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NORMAL SHOCK WAVE DIFFRACTION FOR MONOATOMIC GASES (LARGE BENDS)

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ABSTRACT

Lighthill has considered the diffraction of a normal shock wave past a small bend for $\gamma = 1.4, \gamma$ *being the ratio of specific heats. Srivastava extended the work of Lighthill to general value of* γ *and several important results for* $\gamma = \frac{5}{3}$ *(monoatomic gases) have been obtained earlier as also most recently. In the present paper pressure distribution over diffracted shock for* $\gamma = \frac{5}{3}$ *using Sakurai and Takayama theory has been obtained and compared with earlier results.*

Keywords: *Diffraction, Large bends, Pressure Distribution, Monoatomic gases, Singular perturbation.*

I. INTRODUCTION

Lighthill (1949) considered the diffraction of a normal shock wave past a small bend for $\gamma = 1.4, \gamma$ being the ratio of specific heats. Srivastava (1963) extended the work of Lighthillfor $\gamma = \frac{5}{3}$. He predicted the pressure distribution over the wall as also the curvature of the diffracted shock wave. Srivastava (2016) has obtained pressure distribution over the diffracted shock wave for $\gamma = \frac{5}{3}$ and $M = 1.36$. $M = \frac{U}{a_0}$ is the Mach number of the shock wave, U is the velocity of shock wave and a_0 is the sound speed ahead of the shock wave.

These results are based on Lighthill's (1949) theory and applicable to smaller bends. Sakurai and Takayama (2005) extended the theory of Lighthill (1949) to second order terms using singular perturbation techniques. The theory of Sakurai and Takayama (2005) covers shock diffraction for larger bends. In the present paper pressure distribution over diffracted shock has been obtained following Sakurai and Takayama (2005) theory for $\gamma = \frac{5}{3}$. Earlier Srivastava (2013) have given results for curvature and vorticity distribution over the diffracted shock both from Lighthill's (1949) theory and Sakurai and Takayama (2005)theory. Srivastava's (1994) book may be used for reference.

Mathematical Formulation: Let there be a normal shock of any strength moving into monoatomic gases suffer diffraction after meeting a small bend of angle δ. Let the velocity, pressure, density and sound speed ahead of shock wave be $0,\ p_0,\ \rho_0,\ a_0$ and behind the shock be $q_1,\ p_1,\ \rho_1,\ a_1$ before diffraction. Then applying the principle of conservation of mass, momentum and energy across the shock, we obtain for $\gamma = \frac{5}{3}$

$$q_1 = \frac{3}{4}U\left(1 - \frac{a_0^2}{U^2}\right) \qquad \text{- (1)}$$

$$\rho_1 = \frac{4\rho_0}{\left(1 + \frac{3a_0^2}{U^{-2}}\right)} \qquad \text{- (2)}$$

$$p_1 = \frac{3}{4}\rho_0\left(U^2 - \frac{a_0^2}{5}\right) \qquad \text{- (3)}$$

$$M_1 = \frac{q_1}{a_1} = \frac{q_1}{\sqrt{\frac{\gamma p_1}{\rho_1}}} = \frac{3\left(M^2 - 1\right)}{\left[(5M^2 - 1)(M^2 + 3)\right]^{1/2}} \qquad \text{- (4)}$$

The wedge is formed of two walls having a small angle δ between them. After diffraction let $\vec{q}_2,\ p_2,\ \rho_2$ and S_2 be the velocity vector, pressure, density and entropy at any point. The flow is two dimensional after diffraction. We take the origin on the leading edge of the wedge, Y axis along the leading edge of the wedge and X axis is along the original wall produced. For this configuration the equations of conservation of mass, momentum and energy can be written as

$$\frac{D\rho_2}{Dt} + \rho_2 \operatorname{div} \vec{q}_2 = 0 \qquad \text{- (5)}$$

$$\frac{D\vec{q}_2}{Dt} + \frac{1}{\rho_2}\nabla \mathrm{p}_2 = 0 \qquad \text{- (6)}$$

$$\frac{DS_2}{Dt} = 0 \qquad \text{- (7)}$$

Now we have introduce the following transformations

$$\frac{X - q_1 t}{a_1 t} = x \qquad -(8)$$

$$\frac{Y}{a_2 t} = y \qquad -(9)$$

$$\frac{\vec{q}_2}{q_2} = (1 + u, v) \qquad -(10)$$

$$\frac{p_2 - p_1}{a_1\ q_1\ p_1} = p \qquad -(11)$$

We assume that $\vec{q}_2, p_2, \rho_2$ differ by small quantities from the values $(q_1, 0)$, p_1, ρ_1 which they had before diffraction, then using the equations (5), (6), (7) and (8), (9), (10), (11) we obtain a single second order differential equation in *p*, namely

$$\nabla^2 p = \left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + 1 \right)\left(x\frac{\partial p}{\partial x} + y\frac{\partial p}{\partial y} \right) \qquad -(12)$$

The characteristics of the differential equation (12) are tangents to the unit circle $x^2 + y^2 = 1$, so that the disturbed region is enclosed by the arc of the unit circle, the diffracted shock and the wedge surface.

The position of the straight portion of the shock wave in *x*, *y* coordinates is given by $x = k$, where $k = \frac{U - q_1}{a_1}$ and the coordinates of the corner is $(-M_1, 0)$. For $\gamma = \frac{5}{3}$, the expression for $k = \left(\frac{M^2 + 3}{5M^2 - 1} \right)^{1/2}$ Lighthill (1949) has worked out a function which satisfies all the boundary conditions.

The function is given by

$$W(z_1) = \frac{\partial p}{\partial y_1} + i\frac{\partial p}{\partial x_1} = \frac{C\delta\left[D(z_1 - x_0 - 1)\right]}{(z_1^2 - 1)^{1/2}\left\{\alpha - i(z_1 - 1)^{1/2}\right\}\left\{\beta - i(z_1 - 1)^{1/2}\right\}} \times \frac{1}{(z_1 - x_0)} \qquad -(13)$$

In (13), $z_1 = x_1 + iy$

In the final z_1 plane, the imaginary part on the left hand side of (13) gives the pressure derivative which determines the pressure distribution over the diffracted shock. If one does that, then we have

$$\frac{\partial p}{\partial x_1} = \frac{C\delta}{\left(x_2^2-1\right)^{1/2}}\left[D-\frac{1}{\left(x_1-x_0\right)}\right]\frac{(\alpha+\beta)\left(x_1-1\right)^{1/2}}{\left[\alpha^2+\left(x_1-1\right)\right]\left[\beta^2+\left(x_1-1\right)\right]} \qquad \text{- (14)}$$

In (14) all the quantities are functions of the Mach number of the shock wave M except x_1 which runs from 1 to ∞ on the diffracted shock in the transformed plane and is connected to y in the physical plane through the relation

$$\frac{y}{k'} = \left(\frac{x_1-1}{x_1+1}\right)^{1/2}, \quad k' = \sqrt{1-k^2} \qquad \text{- (15)}$$

The wall is given by $y=0$ so from (15) $\frac{y}{k'}=0$ on the wall. From (15), therefore $x_1 = 1$ for $\frac{y}{k'}=0$. Further at the intersection of the unit circle and shock wave $y=k'$, so that at this point $\frac{y}{k'}=1$. From this to be true $x_1 \to \infty$ from (15). The theory of Lighthill (1949) was extended by Sakurai and Takayama (2005) to higher δ by considering second order terms through singular perturbation technique. Sakurai and Takayama (2005) assumed y on the diffracted shock and computed $\bar{y}$ from their extended theory and then computed $\bar{x}$. The relationship between $\bar{y}$ and $\bar{x}$ is the same as given by (15) in which y is replaced by $\bar{y}$ and x_1 by $\bar{x}_1$. The new $\bar{y}$ and $\bar{x}$ are used to calculate pressure distribution from equation (14). The modified results of Sakurai and Takayama (2005) required for calculations are given below

$$\bar{y} = \sqrt{r^2-k^2}, \quad r = \xi + \delta r_1, \qquad \xi = \sqrt{y^2+k^2}$$

$$r_1 = \kappa(\phi)\bar{R}\log\bar{R}, \qquad \bar{R} = \left[\rho^2 + \frac{2\bar{X}_0 k\rho}{\xi} + X_0^2\right]^{1/2}$$

$$\rho = \frac{1-\sqrt{1-\xi^2}}{\xi}, \qquad \phi = \tan^{-1}\frac{y}{M_1+k} \qquad \text{- (16)}$$

$$\kappa(\phi) = \frac{1}{\pi}\cdot\frac{M_1^4}{1-\sqrt{1-M_1^2}}\cdot\frac{1}{\cos\phi\cos 2\phi}\left[1+\frac{\gamma+1}{2}\cdot\frac{M_1^2}{1-M_1^2}\cdot\cos 2\phi\right]$$

where ϕ in $\kappa(\phi)$ is a variable.

$$X_0^2 = \frac{1-\sqrt{1-M_1^2}}{M_1}$$

y is the y coordinate on the diffracted shock and ξ is the strained variable and other variables are connected within themselves.

Numerical Calculations: In my earlier paper (2016) pressure distribution has been obtained by integrating equation (14). The pressure p being known at $x_1 = \infty (\bar{x}_1 = \infty)$, the point at the intersection of shock and unit circle, the pressure at $y/k' = 0,\ y/k' = 0.2,\ y/k' = 0.4,\ y/k' = 0.6,\ y/k' = 0.8,\ y/k' = 1$ wasobtained and a Table-1 was prepared. We reproduce the Table-1 here for a comparative study with the results of present paper. In the present paper p has been obtained at $y/k' = 0,\ y/k' = 0.2,\ y/k' = 0.4,\ y/k' = 0.6,\ y/k' = 0.8,\ y/k' = 1$. Theresults are in Table-2. The equation (14) and (15) has been used for obtaining the results.

Table-1

y/k'	0	0.2	0.4	0.6	0.8	1
$-p/k\delta$	3.82	3.35	2.995	2.41	1.58	0

Table-2

y/k'	0	0.2	0.4	0.6	0.8	1
$-p/k\delta$	3.82	3.36	2.999	2.42	1.62	0

The trend of $-p/k\delta$ for higher δ remains the same as that of for lower δ. The values are slightly higher in case of higher δ.

II. CONCLUSION

The results demonstrated here are both from the Lighthill's theory (1949) for lower δ and the results for higher δ from Sakurai and Takayama's theory (2005). These are substantial calculations and will find use in the field of aeronautics.

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