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AN INVESTIGATION ON LOCAL STABILITY AND GLOBAL STABILITY OF A SIGNIFICANT THREE-SPECIES ECOLOGICA MODEL

G. Basava Kumar[1], K.V.L.N.Acharyulu[2], M.N.Srinivas[3]

[1] *Faculty, Dept. of Mathematics,*

Gokula Krishna College of Engineering, Sullurpeta-524121, Andra Pradesh, India,.

[2]*Faculty,Dept. of Mathematics,*

Bapatla Engineering College (Autonomous),Bapatla-522102, AndraPradesh, India,.

[*]Corresponding Author Email: dr.kvlna@gmail.com

[3]*Faculty, Department of Mathematics,Vellore Institute of Technology (VIT)*

Vellore-632 014, Tamilnadu ,India

ABSTRACT:

In this Paper, the three species ecosystem is thoroughly discussed by constructing the model with a Prey, a Predator and an Enemy (Super Predator) to the Ammensal Prey. Limited resources are taken for three species. Perturbation analysis is applied to observe the nature of the system. Local stability and Global stability are examined.Series solutions are determined by employing Homotopy perturbation method. The numerical graphs are illustrated.

Key words: *Ammensal, Enemy Species, Stability, Local Stability, Global Stability, Routh-Huruwitzcriterion ,HPM.*

1. INTRODUCTION:

Mathematical ecology describes how for centuries different types of plants, animals and human beings will survive together in the same place by sharing available resources. Ecology may also be referred as the study of the diversity and distribution of various organisms utilizing the same resources in the same habitat. The Local stability and Global stability of ecological ammensalism was analyzed by K.V.L.N.Acharyulu and N.ch. PattabhiRamacharyulu in diverse aspects [6-8]. Many Researchers [1-5] and mathematicians[9-10] have established valuable principles to examine the behavior of various ecological models.

1.1 Notations Adopted for three species:

N_1 (t) :The population of the Prey-Ammansal Species.

N_2 (t):The population of the predator striving of the prey N_1

N_3 (t):The Population of the enemy to the prey N_1

a_i :The natural growth rates of N_i, i = 1,2,3

a_{ii}:The rate of decrease of N_i due to insufficient resources of N_i,i =1,2,3

a_{12}:The rate of decrease of the prey (N_1) due to inhibition bythe predator (N_2)

a_{32}:The rate of increase of the predator (N_2) due to its successfulpromotion by enemy(N_3)

a_{21}:The rate of increase of the predator (N_2) due to its successfulattacks on the prey (N_1)

K_i :a_i/ a_{ii}: Carrying capacities of N_i ,i = 1, 2, 3.

α :a_{12}/a_{11}: Co-efficient of Ammensalism.

β :a_{21}/a_{22}:Co-efficient of prey inhibition (suffering)

γ :a_{32}/a_{33}:Co-efficient of predator consumption of the prey.

All the model parameters assumed to be non-negative constants.

2 .BASIC EQUATIONS:

The classical equations for a three species multi-interactive ecosystem arespecified by the resulting system of non-linear ordinary differential equations.

(i) The equation for the growth rate of Prey species (N_1):

$$\frac{dN_1}{dt} = a_{11}N_1(K_1 - N_1 - \alpha N_2) \quad (1)$$

(ii) The equation for the growth rate of predator species (N_2):

$$\frac{dN_2}{dt} = a_{22} N_2 (K_2 - N_2 - \beta N_1) \quad (2)$$

(iii) The equation for the growth rate of Super Predator species (N_3):

$$\frac{dN_3}{dt} = a_{33} N_3(K_3 - N_3 + \gamma N_2) \quad (3)$$

Here The co-existent state or normal steady state $E^*\left(\overline{N_1},\overline{N_2},\overline{N_3}\right)$ occurs at

(viii) $$\overline{N_1} = \frac{K_1 - \alpha K_2}{1-\alpha\beta}; \quad \overline{N_2} = \frac{K_2 - \beta K_1}{1-\alpha\beta}; \quad \overline{N_3} = \left(\frac{K_3(1-\alpha\beta) - \gamma(K_2 - \beta K_1)}{1-\alpha\beta}\right) \quad (4)$$

Where $E^*\left(\overline{N_1},\overline{N_2},\overline{N_3}\right) > 0$ If $K_1 > PK_2 + \alpha K_3$ & $K_3 > K_1/\alpha Q$

3.ANALYSIS OF STABILITY AT CO-EXISTENT STATE:

Stability Analysis in terms of Local and Global Stability of the considered system in view of Routh-Hurwitz criteria, Lyapunov Theorem, Diffusive analysis, Stochastic Analysis is discussed with the following theorems.

Theorem (3.1):

$If\ \Delta = \begin{bmatrix} -a_{11}\overline{N_1} & -\alpha a_{11}\overline{N_1} & 0 \\ -\beta a_{21}\overline{N_2} & -a_{22}\overline{N_2} & 0 \\ 0 & \gamma a_{33}\overline{N_3} & -a_{33}\overline{N_3} \end{bmatrix}$ s a Jocbian matrix with the valid conditions

$K_1 - \overline{N_1} - \alpha\overline{N_2} = 0, K_2 - \overline{N}_2 - \beta\overline{N_1} = 0, K_3 - \overline{N_3} + \gamma\overline{N_2} = 0$ then the corresponding system is Locally stable at Coexistence Equilibrium State $E^*\left(\overline{N_1}, \overline{N_2}, \overline{N_3}\right)$

Proof:

The characteristic equation of A is $|\Delta - \lambda I| = 0$

$$\text{i.e} \begin{vmatrix} -a_{11}\overline{N_1} - \lambda & -\alpha a_{11}\overline{N_1} & 0 \\ -\beta a_{21}\overline{N_2} & -a_{22}\overline{N_2} - \lambda & 0 \\ 0 & \gamma a_{33}\overline{N_3} & -a_{33}\overline{N_3} - \lambda \end{vmatrix} = 0$$

$$\lambda = \left(-a_{33}\overline{N_3} - \lambda\right)\left[\left(\lambda + a_{11}\overline{N_1}\right)\left(\lambda + a_{22}\overline{N_2}\right) + a_{12}a_{21}\alpha\beta\overline{N_1}\,\overline{N_2}\right] = 0$$

$$i.e\ \lambda = \lambda^3 + \lambda^2\left(a_{11}\overline{N_1} + a_{22}\overline{N_2} + a_{33}\overline{N_3}\right) + \lambda\left[\overline{N_1}\,\overline{N_2}a_{11}a_{22}(1-\alpha\beta) + a_{22}a_{33}\overline{N_2}\,\overline{N_3} + a_{11}a_{33}\overline{N_1}\,\overline{N_3}\right] +$$

$$\left(a_{11}a_{22}a_{33}\overline{N_1}\,\overline{N_2}\,\overline{N_3}(1-\alpha\beta)\right)$$

By arranging Routh Array, all the elements in the first column are positive.

$Those\ are\ 1 > 0, X_1 > 0, \frac{X_1X_2 - 1.X_3}{X_1} > 0\ \&\ X_3 > 0$ provided $1 - \alpha\beta > 0 \quad \Rightarrow a_{11}a_{22} < a_{12}a_{21}$

where $X_1 = a_{11}\overline{N_1} + a_{22}\overline{N_2} + a_{33}\overline{N_3}$

$X_2 = \left[\overline{N_1}\,\overline{N_2}a_{11}a_{22}(1-\alpha\beta) + a_{22}a_{33}\overline{N_2}\,\overline{N_3} + a_{11}a_{33}\overline{N_1}\,\overline{N_3}\right]$, $X_3 = \left(a_{11}a_{22}a_{33}\overline{N_1}\,\overline{N_2}\,\overline{N_3}(1-\alpha\beta)\right)$

Hence, By Routh-Hurwitz criterion, the system is Locallystable $E^*\left(\overline{N_1}, \overline{N_2}, \overline{N_3}\right)$

Theorem(3.2): The positive equilibrium $E^*\left(\overline{N_1},\overline{N_2},\overline{N_3}\right)$ of system (1)-(3) is globally-asymptotically -stable if $V'(t)<0$

where $V(t)=\left(N_1-\overline{N_1}-\overline{N_1}\ln\left(\frac{N_1}{\overline{N_1}}\right)\right)+l_1\left(N_2-\overline{N_2}-\overline{N_2}\ln\left(\frac{N_2}{\overline{N_2}}\right)\right)+l_2\left(N_2-\overline{N_2}-\overline{N_2}\ln\left(\frac{N_2}{\overline{N_2}}\right)\right)$,

where $l_1,l_2>0$.

Proof: The periodderived of the constructive definite function $V(t)$ is considered to verify the global stability behavior of the interior equilibrium point $E^*\left(\overline{N_1},\overline{N_2},\overline{N_3}\right)$ by using suitable Lyapunov function.

The positive equilibrium $\mathrm{E}^*\left(\overline{\mathrm{N}_1},\overline{\mathrm{N}_2},\overline{\mathrm{N}_3}\right)$

$$V(t)=\left(N_1-\overline{N_1}-\overline{N_1}\ln\left(\frac{N_1}{\overline{N_1}}\right)\right)+l_1\left(N_2-\overline{N_2}-\overline{N_2}\ln\left(\frac{N_2}{\overline{N_2}}\right)\right)+l_2\left(N_2-\overline{N_2}-\overline{N_2}\ln\left(\frac{N_2}{\overline{N_2}}\right)\right),$$

where $l_1,l_2>0$

$$\frac{dV}{dt}=\left(\frac{N_1-\overline{N_1}}{N_1}\right)\frac{dN_1}{dt}+l_1\left(\frac{N_2-\overline{N_2}}{N_2}\right)\frac{dN_2}{dt}+l_2\left(\frac{N_3-\overline{N_3}}{N_3}\right)\frac{dN_3}{dt}$$

$$\frac{dV}{dt}=(N_1-\overline{N_1})\,(a_{11}(K_1-N_1-\alpha N_2)+l_1(N_2-\overline{N_2})\,(a_{22}(K_2-N_2-\beta N_1)+l_2(N_3-\overline{N_3})\,(a_{33}(K_3-N_3+\gamma N_2)$$

$$\frac{dV}{dt}\le-\left[(a_{11}+\frac{\alpha a_{11}}{2}+\frac{\beta}{2})(N_1-\overline{N_1})^2+(1+\frac{\alpha a_{11}}{2}+\frac{\beta}{2}-\frac{\gamma}{2})(N_2-\overline{N_2})^2+(1-\frac{\gamma}{2})(N_3-\overline{N_3})^2\right]$$

$$\Rightarrow\frac{dV}{dt}\le0\ \text{Provided}\ \ 1+\frac{\alpha a_{11}}{2}+\frac{\beta}{2}>\frac{\gamma}{2}\ \text{and}\ 1>\frac{\gamma}{2}\Rightarrow a_{32}<2a_{33}$$

Clearly, $V'(t)<0$, hence the non-diffusive system (1)-(3) is globally asymptotically stable by Lyapunov Theorem

4. SERIES SOLUTIONS BY HOMOTOPY PERTURBATION METHOD:

Let us take the nonlinear differential equation:

$$Au-fr=0,\quad r\in\Omega \qquad (5)$$

With the stationary condition

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$$B\left(u, \frac{\partial u}{\partial n}\right) = 0 \quad , r \in \Gamma$$

Where A is a normal differential operator, B a boundary operator, $f\,r$ is a known analytic function, Γ is the boundary of the domain Ω and $\frac{\partial}{\partial n}$ denotes differentiation along the normal drawn outwards from Ω .

In general the operator A, is divided into two parts: a linear part L and a nonlinear part N.

Therefore above differential equation is expressed in the form of A

$$Lu - Nu - fr = 0 \qquad (6)$$

With the help of Homotopy Perturbation Method (HPM), one can constitute a homotopy

$v\,r, p : \Omega \times 0,1 \rightarrow R$ whichsatisfies

$$H\,v, p = 1 - p(L\,v - L\,u_0) + p(A\,v - f\,r) = 0 \;\; p \in 0,1 \;\; r \in \Omega \qquad (7)$$

It is nothing but

$$H\,v, p = (L\,v - L\,u_0) + pL\,u_0 + p(N\,v - f\,r) = 0 \qquad (8)$$

Where $p \in 0,1$ is named as an embedding parameter,and u_0 is an initial approximation of equation(6.1),which satisfies the boundary conditions

Then equations (7), (8) follow that

$H\,v, 0 = (L\,v - L\,u_0) = 0$ and $H\,v, 1 = (A\,v - f\,r) = 0$

Thus the changing process of p from zero to unity is just that of v, r, p from u_0, r to u, r .

According to the HPM, we can first use the imbedding parameter p as a 'small parameter' and assume that the solutions of the equations (7) and (8) can be written as a power series in p

$$v = v_0 + pv_1 + p^2 v_2 + p^3 v_3 + p^4 v_4 + p^5 v_5 + \ldots$$

The approximate solution of equation (6.1) can be obtained as

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + v_3 + v_4 + v_5 + \ldots$$

Basic Equations of the given ecological model is

$$\frac{dN_1}{dt} = a_{11}N_1 (K_1 - N_1 - \alpha N_2) \quad (9)$$

$$\frac{dN_2}{dt} = a_{22}N_2 (K_2 - N_2 - \beta N_1) \quad (10)$$

$$\frac{dN_3}{dt} = a_{33} N_3 (K_3 - N_3 + \gamma N_2) \quad (11)$$

With initial conditions $N_1(0)=c_1, \& N_2(0)=c_2 \;\; N_3(0)=c_3$

The following system can be constructed by the concept of homotopy as follows

$$v_1^{1} - N_{10}^{1} + p(N_{10}^{1} - a_1v_1 - a_{11}v_1^2 - a_{12}v_1v_2) = 0 \quad (12)$$

$$v_2^{1} - N_{20}^{1} + p(N_{20}^{1} - a_2v_2 - a_{22}v_2^{2} - a_{22}v_1v_2) = 0 \quad (13)$$

$$v_3^{1} - N_{30}^{1} + p(N_{30}^{1} - a_3v_3 - a_{33}v_3^2 + a_{33}v_2v_3) = 0 \quad (14)$$

The initial approximations are considered as

$$v_{1,0}(t) = N_{10}(t) = v_1(0) = c_1 \quad (15)$$

$$v_{2,0}(t) = N_{20}(t) = v_2(0) = c_2 \quad (16)$$

$$v_{3,0}(t) = N_{30}(t) = v_3(0) = c_3 \, (17)$$

And

$$v_1(t) = v_{1,0}(t) + pv_{1,1}(t) + p^2v_{1,2}(t) + p^3v_{1,3}(t) + p^4v_{1,4}(t) + \ldots \quad (17)$$

$$v_2(t) = v_{2,0}(t) + pv_{2,1}(t) + p^2v_{2,2}(t) + p^3v_{2,3}(t) + p^4v_{2,4}(t) + \ldots \quad (18)$$

$$v_3(t) = v_{3,0}(t) + pv_{3,1}(t) + p^2v_{3,2}(t) + p^3v_{3,3}(t) + p^4v_{3,4}(t) + \ldots \quad (19)$$

The 4-terms approximations are sufficient, we get

$$N_1(t) = \lim_{p\to 1} v_1(t) = \sum_{x=0}^{4} v_{1,x}(t) = v_{1,0}(t) + p\,v_{1,1}(t) + p^2v_{1,2}(t) + p^3v_{1,3}(t) + p^4v_{1,4}(t)$$

$$N_2(t) = \lim_{p\to 1} v_2(t) = \sum_{x=0}^{4} v_{2,x}(t) = v_{2,0}(t) + p\,v_{2,1}(t) + p^2v_{2,2}(t) + p^3v_{2,3}(t) + p^4v_{2,4}(t)$$

$$N_3(t) = \lim_{p\to 1} v_2(t) = \sum_{x=0}^{4} v_{3,x}(t) = v_{3,0}(t) + p\,v_{3,1}(t) + p^2v_{3,2}(t) + p^3v_{3,3}(t) + p^4v_{3,4}(t)$$

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The convergent series solutions are obtainedby HPM as

$$N_1(t)=c_1+(a_1-a_{11}c_1-a_{12}c_2)c_1t$$

$$+[(a_1+2a_{11}c_1+a_{12}c_2)c_1+a_{12}c_1c_2(a_2-a_{22}c_2-a_2c_1)(a_1-a_{11}c_1-a_{12}c_2)]\frac{t^2}{2}$$

$$[(a_1+2a_{11}c_1+a_{12}c_2)((a_1+2a_{11}c_1+a_{12}c_2)c_1+a_{12}c_1c_2(a_2-a_{22}c_2-a_2c_1)(a_1-a_{11}c_1-a_{12}c_1))$$
$$+(a_2-a_{22}c_2-a_{21}c_1)(a_2+2a_{22}c_2+a_{21}c_1)c_2++a_{21}c_1c_2(a_1-a_{11}c_1-a_{12}c_2)$$
$$+2(a_1-a_{11}c_1-a_{12}c_2)+a_{11}(a_1-a_{11}c_1-a_{12}c_2)+c_1c_2(a_1-a_{11}c_1-a_{12}c_1)]\frac{t^3}{6}$$

$$[[(a_1+2c_1a_{11}+2c_2a_{12})((a_2+2a_{22}c_2+a_{21}c_1)(a_2-a_{22}c_2-a_2c_2)(a_1+2a_{11}c_1+a_{12}c_2)$$
$$+a_{12}c_1c_2(a_1-a_{11}c_1-a_{12}c_1))+2(a_1-a_{11}c_1-a_{12}c_2)+a_{11}(a_1-a_{11}c_1-a_{12}c_2)+c_1c_2(a_1-a_{11}c_1-a_{12}c_1)]$$
$$+6a_{11}[(a_1-a_{11}c_1-a_{12}c_2)^2c_1^{\,2}(a_1+2c_1a_{11}+2c_2a_{12})+a_{12}c_1c_2(a_1-a_{11}c_1-a_{12}c_1)]$$
$$+a_{21}c_1((a_1+2a_{11}c_1+a_{12}c_2)+a_{12}(a_2-a_{22}c_2-a_2c_2)c_1c_2)$$
$$+2c_2(a_1-a_{11}c_1-a_{12}c_1)((a_2-a_{22}c_2-a_2c_2)c_2a_{22}+a_{21}c_1(a_1-a_{11}c_1-a_{12}c_2))$$
$$+3a_{12}((a_1-a_{11}c_1-a_{12}c_2)(a_1-a_{11}c_1-a_{12}c_1)(a_2+2a_{22}c_2+a_{21}c_1)c_1c_2+a_{21}c_1(a_1-a_{11}c_1-a_{12}c_1))$$
$$+(a_1-a_{11}c_1-a_{12}c_2)(a_2-a_{22}c_2-a_{21}c_1)(a_1+2a_{11}c_1+a_{12}c_2)c_1c_2+a_{12}c_1c_2(a_2-a_{22}c_2-a_{21}c_1))]\frac{t^4}{24}$$

$$N_2(t)=c_2+(a_2-a_{22}c_2-a_{21}c_1)c_2t$$

$$+[(a_2-a_{22}c_2-a_{21}c_1)(a_2+2a_{22}c_2+a_{21}c_1)c_2+a_{21}c_1c_2(a_1-a_{11}c_1-a_{21}c_1)]\frac{t^2}{2}$$

$$[(a_2+2a_{22}c_2+a_{21}c_1)((a_2+2a_{22}c_2+a_{21}c_2)(a_2-a_{22}c_2-a_2c_1)c_2+a_{21}c_1c_2(a_2-a_{22}c_2-a_2c_1)$$
$$+a_{21}c_2c_1(a_1-a_{11}c_1-a_{12}c_2)(a_1+2a_{11}c_1+a_{12}c_2)++a_{12}c_1c_2(a_2-a_{22}c_2-a_2c_1)$$
$$+2a_{22}c_2^{\,2}(a_2-a_{22}c_2-a_2c_1)^2+a_{21}c_1(a_1-a_{11}c_1-a_{12}c_2)]\frac{t^3}{6}$$

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$$[[(a_2+2a_{22}c_2+a_{21}c_2)((a_2+2a_{22}c_2+a_{21}c_1)(a_2-a_{22}c_2-a_{21}c_1)(a_2+2a_{22}c_2+a_{21}c_2)c_2$$
$$+a_{21}c_1c_2(a_1-a_{11}c_1-a_{12}c_1))+a_{21}c_2c_1(a_1-a_{11}c_1-a_{12}c_2)(a_1+2c_1a_{11}+c_2a_{12})+a_{12}(a_2-a_{22}c_2-a_{21}c_1)c_1c_2$$
$$+2c_2(a_2-a_{22}c_2-a_{21}c_1)((a_2-a_{22}c_2-a_{21}c_1)c_2a_{22}+a_{21}c_1(a_1-a_{11}c_1-a_{12}c_2))$$
$$+a_{21}c_2((a_1+2a_{11}c_1+a_{12}c_2)(a_1-a_{11}c_1-a_{12}c_2)(a_1+2c_1a_{11}+c_2a_{12})c_1+a_{12}(a_2-a_{22}c_2-a_{21}c_1)c_1c_2$$
$$+(a_2-a_{22}c_2-a_{21}c_1)(a_2+2a_{22}c_2+a_{21}c_1)c_2+a_{21}(a_2-a_{22}c_2-a_{21}c_1)c_1c_2$$
$$+2c_2(a_1-a_{11}c_1-a_{12}c_2)((a_1-a_{11}c_1-a_{12}c_2)a_{11}+(a_2-a_{22}c_2-a_{21}c_1)c_1c_2)$$
$$+6a_{22}((a_2-a_{22}c_2-a_{21}c_1)(a_2-a_{21}c_2-a_{22}c_2)(a_2+2a_{22}c_2+a_{21}c_1)c_1c_2+a_{21}c_1c_2(a_1-a_{11}c_1-a_{12}c_1))$$
$$+3a_{21}(a_1-a_{11}c_1-a_{12}c_2)(a_2-a_{22}c_2-a_{21}c_1)(a_2+2a_{22}c_2+a_{21}c_1)c_1c_2+a_{21}c_1c_2(a_1-a_{11}c_1-a_{21}c_1))$$
$$+3a_{21}(a_1-a_{11}c_1-a_{12}c_2)(a_2-a_{22}c_2-a_{21}c_1)(a_1+2a_{11}c_1+a_{12}c_2)c_1c_2+a_{12}c_1c_2(a_2-a_{22}c_2-a_{21}c_1))]\frac{t^4}{24}$$

$$N_3(t)=c_3+(a_3-a_{33}c_3+a_{32}c_2)c_3t$$
$$+[(a_3-a_{33}c_3+a_{32}c_2)((a_3+2a_{33}c_3-a_{32}c_2)c_3-a_{32}((a_2-a_{22}c_2-a_{21}c_1)c_2c_3))]\frac{t^2}{2}$$
$$[(a_3+2a_{33}c_3-a_{32}c_2)(a_3-a_{33}c_3+a_{32}c_2)(a_3+2a_{33}c_3-a_{33}c_2)c_3-a_{32}(a_2-a_{22}c_2-a_{21}c_1)c_1c_3$$
$$+a_{21}(a_1-a_{11}c_1-a_{12}c_2)+2(a_3-a_{33}c_3+a_{32}c_2)(a_{33}(a_3-a_{33}c_3+a_{32}c_2-a_{32}(a_2-a_{22}c_2-a_{21}c_1)c_3c_2))]\frac{t^3}{6}$$
$$[[(a_3+2a_{33}c_3-a_{32}c_2)((a_3+2a_{33}c_3-a_{32}c_2)^2(a_3-a_{33}c_3+a_{32}c_2)c_3-a_{32}c_1c_2(a_2-a_{22}c_2-a_{21}c_1)$$
$$+a_{21}c_1c_2(a_2-a_{22}c_2-a_{21}c_1)+2(a_3-a_{33}c_3+a_{32}c_2)(a_{33}(a_3-a_{33}c_3+a_{32}c_2)-a_{32}(a_2-a_{22}c_2-a_{21}c_1))$$
$$-a_{32}c_3(a_2+2a_{22}c_2+a_{21}c_2)((a_2-a_{22}c_2-a_{21}c_1)(a_2+2a_{22}c_2+a_{21}c_2)c_2+a_2c_1c_2(a_1-a_{11}c_2-a_{21}c_1)$$
$$+a_{21}c_1c_2(a_1-a_{11}c_1-a_{12}c_2)(a_1+2a_{11}c_1+a_{12}c_2)+a_{12}c_1c_2(a_2-a_{22}c_2-a_{21}c_1))$$
$$+2c_1c_2(a_{22}(a_2-a_{22}c_2-a_{21}c_1)^2+a_{21}(a_1-a_{11}c_1-a_{12}c_2)(a_2-a_{22}c_2-a_{21}c_1)) \qquad \textbf{5.}$$
$$+6a_{33}((a_3-a_{33}c_3+a_{32}c_2)^2c_3{}^2(a_3+2a_{33}c_3-a_{32}c_2)-a_{32}c_2c_3(a_2-a_{22}c_2-a_{21}c_1)$$
$$-3a_{32}(c_2c_3((a_3-a_{33}c_3+a_{32}c_2)(a_2-a_{22}c_2-a_{21}c_1)(a_2+2a_{22}c_2+a_{21}c_1)+a_{21}c_1c(a_1-a_{11}c_1-a_{21}c_1)_2)$$
$$-3a_{32}c_2c_3((a_2-a_{22}c_2-a_{21}c_1)(a_3-a_{33}c_3+a_{32}c_2)(a_3+2a_{33}c_3-a_{32}c_2)-a_{32}(a_2-a_{22}c_2-a_{21}c_1)))]\frac{t^4}{24}$$

Computer Simulations:

For verification of our results from the above segments, here we give some numerical reproductions using Matrix Laboratory.

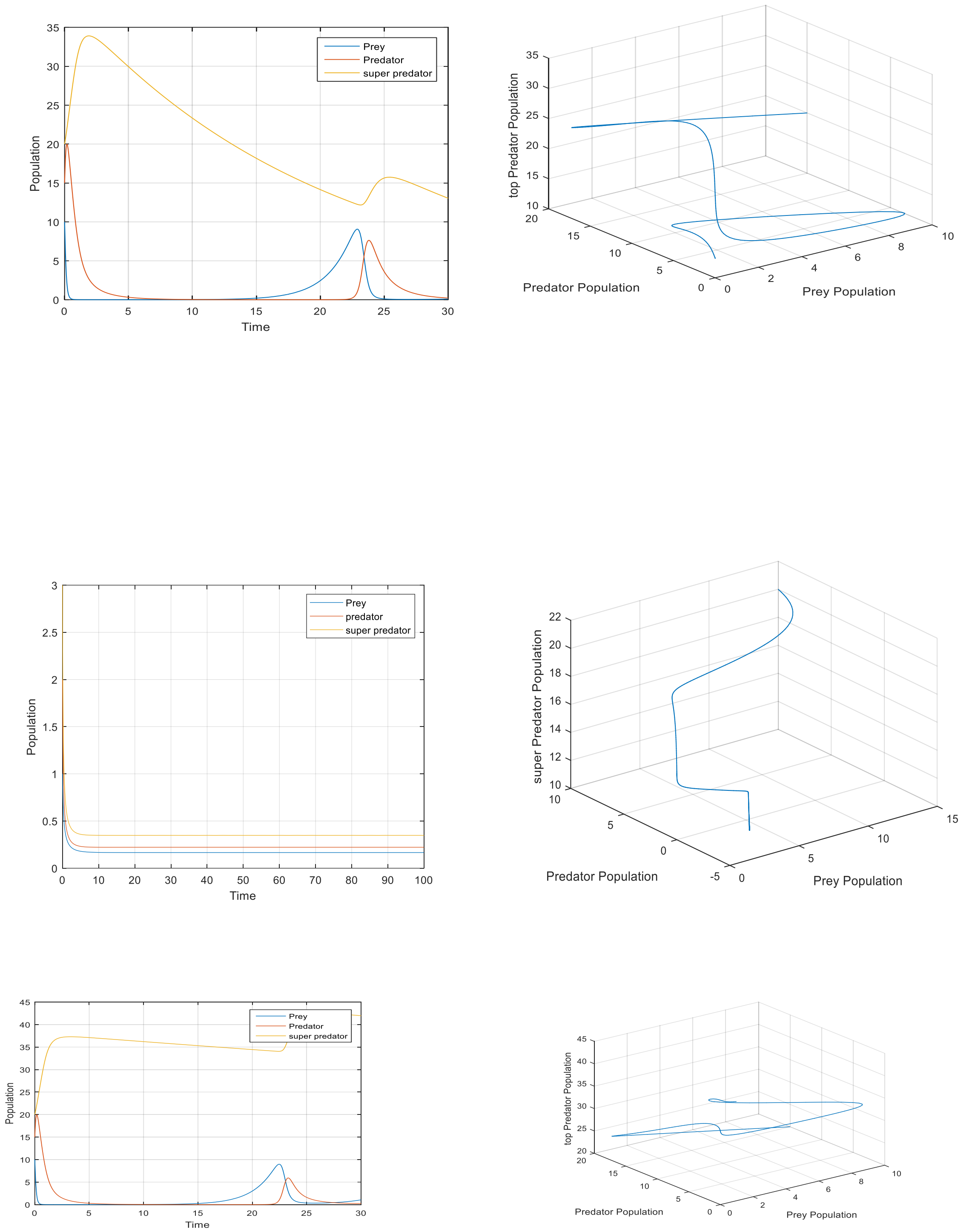

Figure (1)

Figure (1) (left) indicates the population growth against period and figure (1) (right) indicates the phase-portrait dynamics ofprey, Predator and Super predator species for the parameters $a_1 = 3.21;\ a_{11} = 0.002;\ a_{12} = 0.521;\ \alpha = 0.023;\ a_2 = 1.621; a_{22} = 0.422; \beta = 30.\gamma = 40$

6.CONCLUSIONS:

An Investigation on a Mathematical Model of Significant Three Species Ecosystem has been thoroughly done and the following conclusions are drawn:

(i).Identified the behavior of the system with perturbation technique .The three stable cases are occurred in co-existence state.

(ii).Geometric interpretation is performed to illustrate the asymptotic stable.

(iii).Local stability is noticed at interior equilibrium state by Routh-Hurwitz criterion

(iv).Global Stability of the system is observed by constructing suitable Lyapunov function.

(vi).The convergent series solutions of this system are derived by Homotopy Perturbation Method.

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