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BIT ERROR RATE ANALYSIS OF HIERARCHICAL PULSE AMPLITUDE MODULATION

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ABSTRACT

The non-uniform signal constellation of hierarchical modulation is used to offer different level of error protection of transmitted information (such as compressed image and video) based on the relative importance, resulting in more efficient communication over noisy channels. For example, the multiple class of bitstream generated by image and video encoder with different priorities can be categorised into high priority (HP) and low priority (LP) substreams The HP substreams can be protected more strongly as compared to LP substream by exploiting the hierarchical modulation. In this paper, exact bit error rate (BER) analysis of two priority substreams, HP and LP mapped to hierarchical Pulse Amplitude Modulation (HPAM) is obtained for Additive White Gaussian Noise (AWGN) channel. The results confirm that BER of HP bitstream can be reduced at the cost of increase BER of LP bitstream for same transmitted power. Thus, using HPAM, more important information can be transmitted over the noisy channel with more reliability as compared to less important information.

Index Terms : Bit Error Rate, Hierarchical Modulation, Hierarchical Pulse Amplitude Modulation, Additive White Gaussian, Noise (AWGN) Channel

I INTRODUCTION

Due to the change in paradigm of transmission of information from wired channel to wireless channels and the need of multimedia transmissions in a connected world, demand of image and video transmission over wireless networks is increasing day by day [1], [2]. In limited available resources such as bandwidth and transmitted power, providing good quality image and video over wireless connected hand held devices is a very challenging task. To efficiently utilize the bandwidth Modulation techniques with higher levels are preferred, but increase in modulation levels result high Bit error rate (BER) as symbols come closer to each other and thus effect of noise will increase. To overcome this problem, the use of non-uniform signal space constellations are suggested to give different degrees of error protection [3]–[5]. The idea of dividing the broadcasted messages into two or more classes and to give every class a different degree of error protection was introduced firstly in [6] and later used in DVB-T where it is known as hierarchical modulation [7].

Generally, Hierarchical QAM is vastly used in the digital video broadcasting and other wireless technologies. To understand the performance of HQAM over noisy channels, the exact mathematical expression of bit error rate will be very fruitful. However, HQAM comprises of two orthogonal Hierarchical Pulse Amplitude Modulation (HPAM). Thus BER error rate investigation of HPAM will simplify the analysis of HQAM. In this paper, two different priority substreams are considered: high prioirity (HP) and low priority (LP). The HP and LP substreams are mapped to form the symbol of HPAM. Then, exact expression of BER of HP and LP bits for HPAM mapped symbols are derived for additive white aaussian noise (AWGN) channel.

This remaining paper is organized as follows: In section II description of Hierarchical PAM is provided. Average energy of HPAM symbol is obtained in section III. Section IV derive the BER of HP and LP substreams. The results are shown in section V. Conclusion and future work are discussed in section VI.

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II. Hierarchical Pulse Amplitude Modulation

Signal space constellation diagram of two priority 4-HPAM is shown in Fig. 1 using gray coded mapping. Each symbol s_m is represented using 2 bits, $b_{m,1}b_{m,0}$. In two priority level 4-HPAM, the data stream is separated into two priority substreams, namely "High Priority (HP)" and "Low Priority (LP)" with priority of HP greater than that of LP. Symbols are formed by taking a bit from each substream. Most significant bit $b_{m,1}$ decides the sign (or phase) of the signal with minimum euclidean distance d_1 in inter-region axis while least significant bit $b_{m,0}$ decides the value of the symbol with inter-symbol minimum separation, d_2 as shown in Fig. 1. Depending upon d_1 and d_2, HP and LP bits can be protected differently. The relative protection of HP and LP bits is measured in terms of α, called modulation parameter, which is defined as

$$\alpha = \frac{d_1}{d_2} \tag{1}$$

The condition $\alpha \geq 1$ controls the relative degree of protection of the MSB bits $b_{m,1}$ and LSB bits $b_{m,0}$. For $\alpha > 1$, HP bits will get high protection while LP bits will get small protection against noise. Where $\alpha = 1$ corresponds to symmetrical constellation and equal protection of each bits.

Due to the non-uniform constellation of HPAM, the average symbol energy and bit error rate (BER) of it will depend on the parameter α. Therefore, theoretical relations of them in term of α is presented in the following subsections

III. Average Symbol Energy

As described earlier, the constellation of HPAM will be non-uniform and will be decided by the α. Therefore, the average symbol energy of HPAM will also be the function of α and will be obtained as following

The average symbol energy, E^s_{avg}, of M-HPAM is given as:

$$E^s_{avg} = \frac{1}{M}\sum_{i=1}^{M} E^s_i \tag{2}$$

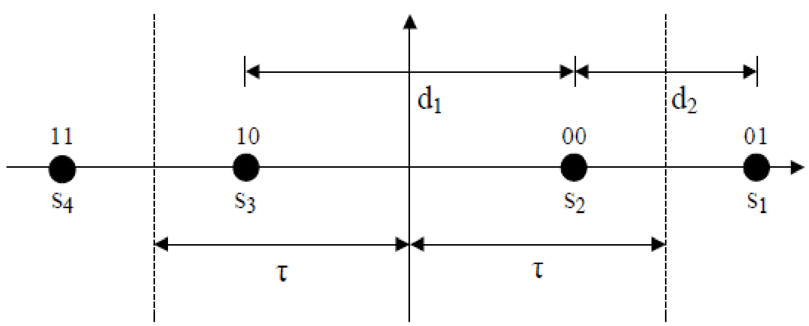

Fig. 1. Constellation diagram of 4 Hierarchical PAM

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where, E_i^s is the energy of the i^{th} symbol, $\forall i = 1, 2, \ldots, M$. However, due to the symmetry across the origin, it is sufficient to calculate the average symbol energy of signals of one side of constellation diagram only. Therefore average symbol energy for 4-HPAM is calculated as:

$$E_{avg}^s = \frac{1}{2}\sum_{i=1}^{2} E_i^s \tag{3}$$

Energy of i^{th} symbol, E_i^s, for 4-HPAM is given by

$$E_i^s = \left(\frac{d_1}{2} + b_{i,0} \cdot d_2\right)^2 \tag{4}$$

where $b_{i,0}$ is the LSB bit of i^{th} symbol which may have value 0 or 1. For 4-HPAM with symmetrical constellation, E_{avg}^s comes out to be

$$\begin{aligned} E_{avg}^s &= \frac{1}{2}\left\{\left(\frac{d_1}{2}\right)^2 + \left(\frac{d_1}{2} + d_2\right)^2\right\} \\ &= \frac{{d_2}^2}{2} k \end{aligned} \tag{5}$$

where

$$k = \left(\frac{\alpha}{2}\right)^2 + \left(\frac{\alpha}{2} + 1\right)^2 \tag{6}$$

Therefore, for a given average symbol energy and modulation parameter α (or k), the euclidean distances among symbols can be designed as

$$d_2 = \sqrt{\frac{2E_{avg}^s}{k}} \tag{7}$$

$$d_1 = \alpha\sqrt{\frac{2E_{avg}^s}{k}} \tag{8}$$

IV. Bit Error Rate (BER) Analysis

The error analysis of a digital communication system is generally carried out in term of bit error rate (BER). The BER analysis of HPAM is presented below which is similar to [8]:

Let the received signal component, r, is

$$r = x + n \tag{9}$$

where x is the transmitted signal component and n is the Additive White Gaussian Noise (AWGN) component with mean , $\mu = 0$ and variance, $\sigma^2 = N_0/2$.

Probability of error for MSB bit, b_1 denoted by $P_e^{b_1}$ is (refer to Fig. 1)

$$P_e^{b_1} = \frac{1}{4}\Big\{P(r < 0|s_1 \text{ was transmitted}) + P(r < 0|s_2 \text{ was transmitted})$$

$$+ P(r > 0|s_3 \text{ was transmitted}) + P(r > 0|s_4 \text{ was transmitted})\Big\} \tag{10}$$

where s_1, s_2, s_3 and s_4 can be found from Fig. 1 as

$$\begin{aligned} s_1 &: x = \frac{d_1}{2} + d_2 \\ s_2 &: x = \frac{d_1}{2} \\ s_3 &: x = -\frac{d_1}{2} \\ s_4 &: x = -\left(\frac{d_1}{2} + d_2\right) \end{aligned} \tag{11}$$

Exploiting the symmetry around origin, the average probability of bit error, $P_e^{b_1}$, can be rewritten as

$$\begin{aligned} P_e^{b_1} &= \frac{1}{2}\Big\{P(r > 0|s_3 \text{ was transmitted}) + P(r > 0|s_4 \text{ was transmitted})\Big\} \\ &= \frac{1}{2}\left\{P\left(r > 0|x = -\frac{d_1}{2}\right) + P\left(r > 0|x = -\left(\frac{d_1}{2} + d_2\right)\right)\right\} \\ &= \frac{1}{2}\left\{P\left(n > \frac{d_1}{2}\right) + P\left(n > \left(\frac{d_1}{2} + d_2\right)\right)\right\} \end{aligned} \tag{12}$$

For AWGN noise with zero mean and variance = σ^2, the probability that $n > d$ is,

$$\begin{aligned} P(n > d) &= \frac{1}{\sigma\sqrt{2\pi}} \int_d^{\infty} e^{-\left(\frac{x^2}{2\sigma^2}\right)} dx \\ &= Q\left(\frac{d}{\sigma}\right) \end{aligned} \tag{13}$$

where $Q(z)$ is a Q-function defined as [9]

$$Q(z) = \frac{1}{\sqrt{2\pi}} \int_z^{\infty} e^{-\left(\frac{x^2}{2}\right)} dx \tag{14}$$

Therefore, using Eqn. (13), the Eqn. (12) can be rewritten as

$$P_e^{b_1} = \frac{1}{2}\left\{Q\left(\frac{d_1}{2\sigma}\right) + Q\left(\frac{\frac{d_1}{2} + d_2}{\sigma}\right)\right\} \tag{15}$$

Replacing the value of $\sigma = \sqrt{N_0/2}$ in Eqn. (15), finally the expression of $P_e^{b_1}$ can be expressed in term of Q-function as

$$\begin{aligned} P_e^{b_1} &= \frac{1}{2}\left\{Q\left(\frac{d_1}{2\sqrt{N_0/2}}\right) + Q\left(\frac{\frac{d_1}{2} + d_2}{\sqrt{N_0/2}}\right)\right\} \\ &= \frac{1}{2}\left\{Q\left(\frac{d_1}{\sqrt{2N_0}}\right) + Q\left(\frac{d_1 + 2d_2}{\sqrt{2N_0}}\right)\right\} \end{aligned} \tag{16}$$

Similarly, with almost same procedure, probability of error in LSB bit , $P_e^{b_0}$, can be obtained as

$$P_e^{b_0} = \frac{1}{2}\left\{2Q\left(\frac{d_2}{\sqrt{2N_0}}\right) + Q\left(\frac{2d_1+d_2}{\sqrt{2N_0}}\right) - Q\left(\frac{2d_1+3d_2}{\sqrt{2N_0}}\right)\right\} \tag{17}$$

Since, b_1, is taken from HP substream and bit, b_0, is taken from LP substream, the probability of bit error for

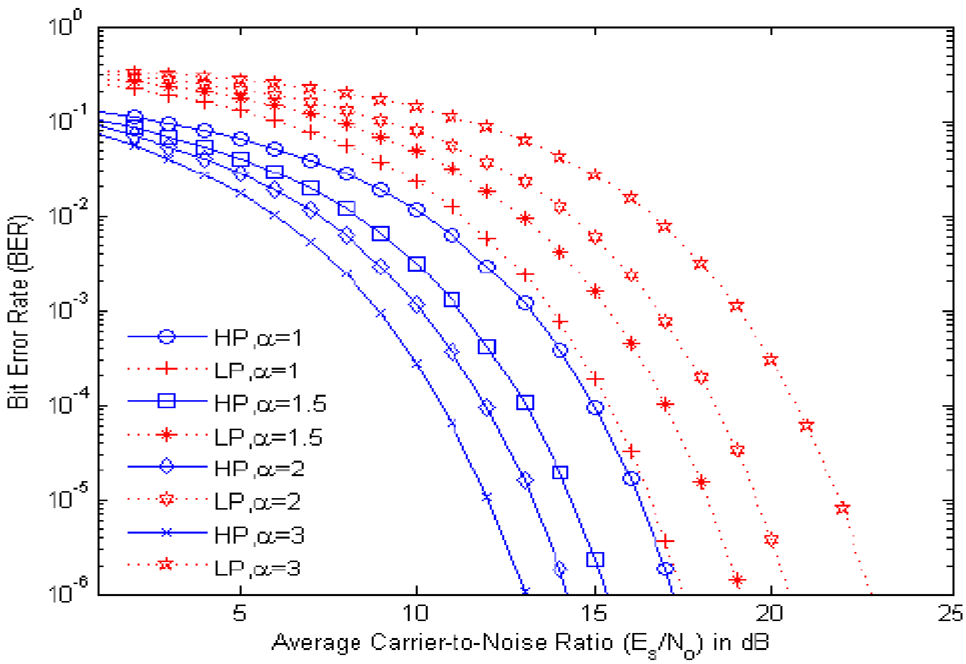

Fig. 2. Bit error rate performance of 4 Hierarchical PAM

HP and LP substreams, denoted as P_e^{HP} and P_e^{LP} respectively, are given as follows

$$P_e^{HP} = P_e^{b_1} = \frac{1}{2}\left\{Q\left(\frac{d_1}{\sqrt{2N_0}}\right) + Q\left(\frac{d_1+2d_2}{\sqrt{2N_0}}\right)\right\} \tag{18}$$

$$P_e^{LP} = P_e^{b_0} = \frac{1}{2}\left\{2Q\left(\frac{d_2}{\sqrt{2N_0}}\right) + Q\left(\frac{2d_1+d_2}{\sqrt{2N_0}}\right) - Q\left(\frac{2d_1+3d_2}{\sqrt{2N_0}}\right)\right\} \tag{19}$$

The above equations can be rewritten in term of $E_{avg}^s/N_0, \alpha$ and k using Eqns. (7) and (8) as

$$P_e^{HP} = \frac{1}{2}\left\{Q\left(\alpha\sqrt{\frac{1}{k}\frac{E_{avg}^s}{N_0}}\right) + Q\left((2+\alpha)\sqrt{\frac{1}{k}\frac{E_{avg}^s}{N_0}}\right)\right\} \tag{20}$$

$$P_e^{LP} = \frac{1}{2}\left\{2Q\left(\sqrt{\frac{1}{k}\frac{E_{avg}^s}{N_0}}\right) + Q\left((2\alpha+1)\sqrt{\frac{1}{k}\frac{E_{avg}^s}{N_0}}\right)\right.$$
$$\left. - Q\left((2\alpha+3)\sqrt{\frac{1}{k}\frac{E_{avg}^s}{N_0}}\right)\right\} \tag{21}$$

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V. Results

For the results, additive white gaussian Noise (AWGN) channel with mean zero and power spectral efficiency of $N_o/2$ is considered. Fig. 2 shows the bit error rate performance of the HP and LP substreams over AWGN channel for different values of α. As the value of α increases, the BER of HP bits decreases while the BER of LP bits increases. For example, as α increases from 1 to 1.5, the BER of HP decreases from 1.14×10^{-2} to 3.18×10^{-3} whereas the BER of LP increases from 2.23×10^{-2} to 4.84×10^{-2}. Fig. 3 shows the bit error rate performance versus α at CNR = 10 dB. From this figure also, it is observed that as the α increases the BER of HP substream decreases whereas the BER of LP substream increases.

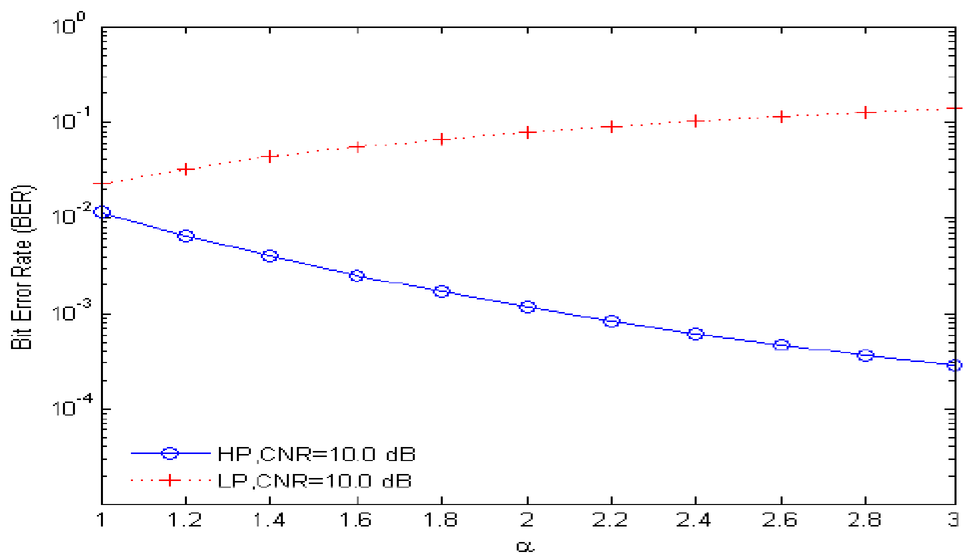

Fig. 3. Bit error rate vs α for CNR = 10 dB for 4 HPAM

VI. Conclusion

In this paper, the two different bitstream namely high priority (HP) and low priority (LP) subtreams are mapped to hierarchical pulse amplitude modulation (HPAM) to form the symbols. Then, the average symbol energy and bit error rate expressions of HP bits and LP bits are obtained and analysed. The hierarchical parameter, α, is used to control the BER rate of HP and LP bitstreams. It was observed that BER of HP substream can be decreased, however, at the cost of increased BER of LP substream by keeping $\alpha > 1$. Thus HPAM can be used to provide unequal error protection scheme for the multimedia data that generate non-uniform importance of information. This investigation will also be helpful in future for analysing the BER of hierarchical Quadrature Amplitude modulation (HQAM) which is used in digital video broadcasting (DVB) standards such as DVB-T and other applications.

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