

ROLE OF VEDIC MATHEMATICS IN DRIVING OPTIMAL SOLUTIONS FOR REAL LIFE PROBLEMS

Deepshikha Bhargava¹, Anita Arya²

^{1,2}AiIT, Department Amity University Rajasthan (India)

ABSTRACT

In this paper, we show the importance of Vedic mathematics to derive the optimal solution for real life problems. Vedic Mathematics is the name given to the ancient system of Mathematics which was rediscovered from the Vedas between 1911 and 1918 by Sri Bharati Krsna Tirthaji (1884-1960). According to his research all of mathematics is based on sixteen Sutras or word-formulae. It is not only useful for arithmetic computations but also helpful for solving problems related to other branches of Mathematics as Algebra, Trigonometry and Calculus etc. Vedic mathematics not only helps in faster computations but also in determining the right solutions to a problem.

Keywords: Algebra, Arithmetic, Calculus, Sutra, and Vedas.

I. INTRODUCTION

Vedic Mathematics is the name given to the ancient system of Mathematics which was rediscovered from the Vedas between 1911 and 1918 by Sri Bharati Krishna Tirthaji (1884-1960). According to his research all of mathematics is based on sixteen Sutras or word-formulae. For example, 'Vertically and crosswise' is one of these Sutras. These formulae describe the way the mind naturally works and are therefore a great help in directing the student to the appropriate method of solution. Instead of learning by repetition, vedic mathematics involves logic and understanding the fundamental concepts. One can do calculations much faster than done by using the conventional method that is taught in schools. It teaches the students the same problem in different ways.

Now, the question is if India has such a wonderful and useful way of calculating mathematical problems then why it is not taught in all the school, right from the beginning. Why we rely just on Western education pattern and learning through logical reasoning instead of mentally solving them. People must be encouraged to learn Vedic mathematics and it must be part of our curricula.

In the Vedic system 'difficult' problems or huge sums can often be solved immediately by the Vedic method. These striking and beautiful methods are just a part of a complete system of mathematics which is far more systematic than the modern 'system'. Vedic Mathematics manifests the coherent and unified structure of mathematics and the methods are complementary, direct and easy.

The simplicity of Vedic Mathematics means that calculations can be carried out mentally (though the methods can also be written down). There are many advantages in using a flexible, mental system. Pupils can invent their own methods, they are not limited to the one 'correct' method. This leads to more creative, interested and intelligent pupils.

Interest in the Vedic system is growing in education where mathematics teachers are looking for something better and finding the Vedic system is the answer. Research is being carried out in many areas including the effects of learning Vedic Math's on children; developing new, powerful but easy applications of the Vedic Sutras in geometry, calculus, computing etc.

But the real beauty and effectiveness of Vedic Mathematics cannot be fully appreciated without actually practicing the system. One can then see that it is perhaps the most refined and efficient mathematical system possible.

II. HISTORY OF VEDIC MATHEMATICS

In all early civilizations, the first expression of mathematical understanding appears in the form of counting systems. Numbers in very early societies were typically represented by groups of lines, though later different numbers came to be assigned specific numeral names and symbols (as in India) or were designated by alphabetic letters (such as in **System in Harappa** Rome). Although today, we take our decimal system for granted, not all ancient civilizations based their numbers on a ten-base system. In ancient Babylon, a sexagesimal (base 60) system was in use.

2.1 The Decimal

In India a decimal system was already in place during the Harappan period, as indicated by an analysis of Harappan weights and measures. Weights corresponding to ratios of 0.05, 0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 50, 100, 200, and 500 have been identified, as have scales with decimal divisions. A particularly notable characteristic of Harappan weights and measures is their remarkable accuracy. A bronze rod marked in units of 0.367 inches points to the degree of precision demanded in those times. Such scales were particularly important in ensuring proper implementation of town planning rules that required roads of fixed widths to run at right angles to each other, for drains to be constructed of precise measurements, and for homes to be constructed according to specified guidelines. The existence of a graduated system of accurately marked weights points to the development of trade and commerce in Harappan society.

2.2 Mathematical Activity in the Vedic Period

In the Vedic period, records of mathematical activity are mostly to be found in Vedic texts associated with ritual activities. However, as in many other early agricultural civilizations, the study of arithmetic and geometry was also impelled by secular considerations. Thus, to some extent early mathematical developments in India mirrored the developments in Egypt, Babylon and China. The system of land grants and agricultural tax assessments required accurate measurement of cultivated areas. As land was redistributed or consolidated, problems of menstruation came up that required solutions. In order to ensure that all cultivators had equivalent amounts of irrigated and non-irrigated lands and tracts of equivalent fertility - individual farmers in a village often had their holdings broken up in several parcels to ensure fairness. Since plots could not all be of the same shape - local administrators were required to convert rectangular plots or triangular plots to squares of equivalent sizes and so on. Tax assessments were based on fixed proportions of annual or seasonal crop

incomes, but could be adjusted upwards or downwards based on a variety of factors. This meant that an understanding of geometry and arithmetic was virtually essential for revenue administrators. Mathematics was thus brought into the service of both the secular and the ritual domains.

2.3 The Indian Numeral System

Although the Chinese were also using a decimal based counting system, the Chinese lacked a formal notational system that had the abstraction and elegance of the Indian notational system, and it was the Indian notational system that reached the Western world through the Arabs and has now been accepted as universal. Several factors contributed to this development whose significance is perhaps best stated by French mathematician, Laplace: "The ingenious method of expressing every possible number using a set of ten symbols (each symbol having a place value and an absolute value) emerged in India. The idea seems so simple nowadays that its significance and profound importance is no longer appreciated. It's simplicity lies in the way it facilitated calculation and placed arithmetic foremost amongst useful inventions."

Brilliant as it was, this invention was no accident. In the Western world, the cumbersome roman numeral system posed as a major obstacle, and in China the pictorial script posed as a hindrance. But in India, almost everything was in place to favor such a development. There was already a long and established history in the use of decimal numbers, and philosophical and cosmological constructs encouraged a creative and expansive approach to number theory. Panini's studies in linguistic theory and formal language and the powerful role of symbolism and representational abstraction in art and architecture may have also provided an impetus, as might have the rationalist doctrines and the exacting epistemology of the Nyaya Sutras, and the innovative abstractions of the Syadvada and Buddhist schools of learning.

Mathematics played a vital role in Aryabhata's revolutionary understanding of the solar system. His calculations on pi, the circumference of the earth (62832 miles) and the length of the solar year (within about 13 minutes of the modern calculation) were remarkably close approximations. In making such calculations, Aryabhata had to solve several mathematical problems that had not been addressed before, including problems in algebra (beej-ganit) and trigonometry (trikonmiti).

Bhaskar I continued where Aryabhata left off, and discussed in further detail topics such as the longitudes of the planets; conjunctions of the planets with each other and with bright stars; risings and settings of the planets; and the lunar crescent. Again, these studies required still more advanced mathematics and Bhaskar I expanded on the trigonometric equations provided by Aryabhata, and like Aryabhata correctly assessed pi to be an irrational number. Amongst his most important contributions was his formula for calculating the sine function which was 99% accurate. He also did pioneering work on indeterminate equations and considered for the first time quadrilaterals with all the four sides unequal and none of the opposite sides parallel.

Another important astronomer/mathematician was Varahamira (6th C, Ujjain) who compiled previously written texts on astronomy and made important additions to Aryabhata's trigonometric formulas. His works on permutations and combinations complemented what had been previously achieved by Jain mathematicians and provided a method of calculation of nCr that closely resembles the much more recent Pascal's Triangle. In the 7th century, Brahmagupta did important work in enumerating the basic principles of algebra. In addition to

listing the algebraic properties of zero, he also listed the algebraic properties of negative numbers. His work on solutions to quadratic indeterminate equations anticipated the work of Euler and Lagrange.

2.4 Emergence of Calculus

In the course of developing a precise mapping of the lunar eclipse, Aryabhata was obliged to introduce the concept of infinitesimals - i.e. *tatkalika gati* to designate the infinitesimal, or near instantaneous motion of the moon, and express it in the form of a basic differential equation. Aryabhata's equations were elaborated on by Manjula (10th C) and Bhaskaracharya (12th C) who derived the differential of the sine function. Later mathematicians used their intuitive understanding of integration in deriving the areas of curved surfaces and the volumes enclosed by them.

2.5 Applied Mathematics, Solutions to Practical Problems

Developments also took place in applied mathematics such as in creation of trigonometric tables and measurement units. Yativrsabha's work *Tiloyapannatti* (6th C) gives various units for measuring distances and time and also describes the system of infinite time measures.

In the 9th C, Mahaviracharya (Mysore) wrote *Ganit Saar Sangraha* where he described the currently used method of calculating the Least Common Multiple (LCM) of given numbers. He also derived formulae to calculate the area of an ellipse and a quadrilateral inscribed within a circle (something that had also been looked at by **Brahmagupta**) The solution of indeterminate equations also drew considerable interest in the 9th century, and several mathematicians contributed approximations and solutions to different types of indeterminate equations.

In the late 9th C, Sridhara (probably Bengal) provided mathematical formulae for a variety of practical problems involving ratios, barter, simple interest, mixtures, purchase and sale, rates of travel, wages, and filling of cisterns. Some of these examples involved fairly complicated solutions and his *Patiganita* is considered an advanced mathematical work. Sections of the book were also devoted to arithmetic and geometric progressions, including progressions with fractional numbers or terms, and formulas for the sum of certain finite series are provided. Mathematical investigation continued into the 10th C. Vijayanandi (of Benares, whose *Karanatilaka* was translated by Al-Beruni into Arabic) and Sripati of Maharashtra are amongst the prominent mathematicians of the century.

The leading light of 12th C Indian mathematics was Bhaskaracharya who came from a long-line of mathematicians and was head of the astronomical observatory at Ujjain. He left several important mathematical texts including the *Lilavati* and *Bijaganita* and the *Siddhanta Shiromani*, an astronomical text. He was the first to recognize that certain types of quadratic equations could have two solutions. His *Chakrawaat* method of solving indeterminate solutions preceded European solutions by several centuries, and in his *Siddhanta Shiromani* he postulated that the earth had a gravitational force, and broached the fields of infinitesimal calculation and integration. In the second part of this treatise, there are several chapters relating to the study of the sphere and its properties and applications to geography, planetary mean motion, eccentric epicyclical model of the planets, first visibilities of the planets, the seasons, the lunar crescent etc. He also discussed astronomical instruments and spherical trigonometry. Of particular interest are his trigonometric equations:

$$\sin(a + b) = \sin a \cos b + \cos a \sin b;$$

$$\sin(a - b) = \sin a \cos b - \cos a \sin b.$$

III. THE SPREAD OF INDIAN MATHEMATICS

The study of mathematics appears to slow down after the onslaught of the Islamic invasions and the conversion of colleges and universities to madrasahs. But this was also the time when Indian mathematical texts were increasingly being translated into Arabic and Persian. Although Arab scholars relied on a variety of sources including Babylonian, Syrian, Greek and some Chinese texts, Indian mathematical texts played a particularly important role. Scholars such as Ibn Tariq and Al-Fazari (8th C, Baghdad), Al-Kindi (9th C, Basra), Al-Khwarizmi (9th C. Khiva), Al-Qayarawani (9th C, Maghreb, author of Kitab fi al-hisab al-hindi), Al-Uqlidisi (10th C, Damascus, author of The book of Chapters in Indian Arithmetic), Ibn-Sina (Avicenna), Ibn al-Samh (Granada, 11th C, Spain), Al-Nasawi (Khurasan, 11th C, Persia), Al-Beruni (11th C, born Khiva, died Afghanistan), Al-Razi (Teheran), and Ibn-Al-Saffar (11th C, Cordoba) were amongst the many who based their own scientific texts on translations of Indian treatises. Records of the Indian origin of many proofs, concepts and formulations were obscured in the later centuries, but the enormous contributions of Indian mathematics was generously acknowledged by several important Arabic and Persian scholars, especially in Spain. Abbasid scholar Al-Gaheth wrote: " India is the source of knowledge, thought and insight". Al-Maoudi (956 AD) who travelled in Western India also wrote about the greatness of Indian science. Said Al-Andalusi, an 11th C Spanish scholar and court historian was amongst the most enthusiastic in his praise of Indian civilization, and specially remarked on Indian achievements in the sciences and in mathematics. Of course, eventually, Indian algebra and trigonometry reached Europe through a cycle of translations, travelling from the Arab world to Spain and Sicily, and eventually penetrating all of Europe. At the same time, Arabic and Persian translations of Greek and Egyptian scientific texts became more readily available in India.

IV. VEDIC MATHEMATICAL SUTRAS

4.1 The sutras

The Vedic Mathematics system uses a collection of sixteen sutras and some sub-sutras. These are given in Sanskrit in the book "Vedic Mathematics". We have Sri Bharati Krsna Tirthaji's English translation for some but not those marked with an asterisk in the list below.

1. By One More than the One Before
2. All from 9 and the Last from 10
3. Vertically and Crosswise
4. Transpose and Apply
5. If the Samuccaya is the same it is Zero
6. If one is in Ratio the other is Zero
7. By Addition and by Subtraction
8. By the Completion or Non-Completion
9. Differential Calculus

10. By the Deficiency
11. Specific and General
12. The Remainders by the Last Digit
13. The Ultimate and Twice the Penultimate
14. By One Less than the One before
15. The Product of the Sums
16. All the Multipliers
17. Proportionately
18. The Remainder Remains Constant
19. The First by the First and the Last by the Last
20. For 7 the Multiplicand is 143
21. By Osculation
22. By Osculation
23. Lessen by the Deficiency
24. Whatever the Deficiency lessen by that amount and set up the Square of the Deficiency
25. Last Totaling 10
26. Only the Last Terms
27. The Sum of the Products
28. By Alternative Elimination and Retention
29. By Mere Observation
30. The Product of the Sum is the Sum of the Products
31. On the Flag

V. APPLICATIONS IN MATHEMATICS

Example1.

A simple idea for factorization of polynomial expressions of two or more variables is rooted in Adyamadyena Sutra—Alternate Elimination and Retention.

Let us consider, a polynomial

$$P(x, y, z) = 2x^2 + 6y^2 + 3z^2 + 7xy + 11yz + 7xz,$$

which can be factorized by setting

$$z = 0:$$

$$P(x, y, 0) = 2x^2 + 7xy + 6y^2 = (2x + 3y)(x + 2y)$$

Now, setting

$y = 0$:

$$P(x, 0, z) = 2x^2 + 7xz + 3z^2 = (2x + z)(x + 3z).$$

By comparing the obtained factorizations (1) and (2) and completing each factor with the additional terms from the Other factorization, we obtain the factorization of $P(x, y, z)$:

$$P(x, y, z) = (2x + 3y + z)(x + 2y + 3z).$$

Also, notice that on substituting

$x = 0$, we obtain

$$P(0, y, z) = 6y^2 + 11yz + 3z^2 = (3y + z)(2y + 3z),$$

in accordance with the Factorization.

Example2.

It is also possible to eliminate two variables at a time. For example, consider the polynomial

$$Q(x, y, z) = 3x^2 + 7xy + 2y^2 + 11xz + 7yz + 6z^2 + 14x + 8y + 14z + 8.$$

Such Eliminations lead to

$$Q(x, 0, 0) = 3x^2 + 14x + 8 = (x + 4)(3x + 2)$$

$$Q(0, y, 0) = 2y^2 + 8y + 8 = (2y + 4)(y + 2)$$

$$Q(0, 0, z) = 6z^2 + 14z + 8 = (3z + 4)(2z + 2).$$

Using a completion method similar to Example 1, we obtain

$$Q(x, y, z) = (x + 2y + 3z + 4)(3x + y + 2z + 2).$$

It is easy to verify that this is indeed a factorization of the polynomial $Q(x, y, z)$.

VI. MAGIC CALCULATIONS

6.1 Converting Kilos to pounds

In this section you will learn how to convert Kilos to Pounds, and Vice Versa.

Let's start off with looking at converting Kilos to pounds. 86 kilos into pounds:

Step-1 multiplies the kilos by two.

To do this, just double the kilos.

$$86 \times 2 = 172$$

Step-2 Divide the answer by ten.

To do this, just put a decimal point one place in from the right.

$$172 / 10 = 17.2$$

Step-3 add step two's answer to step one's answer.

$$172 + 17.2 = 189.2$$

$$86 \text{ Kilos} = 189.2 \text{ pounds}$$

6.2 Temperature Conversions

This is a shortcut to convert Fahrenheit to Celsius and vice versa

Fahrenheit to Celsius and vice versa: Take 30 away from the Fahrenheit, and then divide the answer by two.

This is your answer in Celsius.

Example: convert 74 Fahrenheit into Celsius

$$74 \text{ Fahrenheit} - 30 = 44. \text{ Then divide by two, } 22 \text{ Celsius.}$$

$$74 \text{ Fahrenheit} = 22 \text{ Celsius.}$$

Celsius to Fahrenheit just do the reverse:

Double it, and then add 30.

Celsius double it, is 60, then add 30 is 90

$$30 \text{ Celsius} = 90 \text{ Fahrenheit}$$

6.3 Distance Conversions

This is a useful method for when travelling between imperial and metric countries and need to know what kilometers to miles are.

The formula to convert kilometers to miles is number of (kilometers / 8) x 5

So lets try 80 kilometers into miles

$$80/8 = 10$$

Multiplied by 5 is 50 miles!

Another example 40 kilometers

$$40 / 8 = 5$$

$$5 \times 5 = 25 \text{ mile}$$

VII. CONCLUSION

After going through the content presented in this report, you may, perhaps, have noted a number of applications of methods of Vedic Mathematics. We are aware that this attempt is only to make you familiar with a few special methods. The methods discussed, and organization of the content here are intended for any reader with some basic mathematical background. That is why the serious mathematical issues, higher level mathematical problems are not taken up in this volume, even though many aspects like four fundamental operations, squaring, cubing, linear equations, simultaneous equations. factorization, etc are dealt with. Many more concepts and aspects are omitted unavoidably, keeping in view the scope and limitations of the present volume.

The present volume, even though introductory, has touched almost all the Sutras and sub-Sutras as mentioned in Swamiji's 'Vedic Mathematics'. Further it has given rationale and proof for the methods. As there is a general opinion that the 'so called Vedic Mathematics is only rude, rote, non mathematical and none other than some sort of tricks', the logic, proof and Mathematics behind the 'the so called tricks' has been explained. An impartial reader can easily experience the beauty, charm and resourcefulness in Vedic Mathematics systems. We feel that the reader can enjoy the diversity and simplicity in Vedic Mathematics while applying the methods against the conventional textbook methods. The reader can also compare and contrast both the methods.

The Vedic Methods enable the practitioner improve mental abilities to solve difficult problems with high speed and accuracy.

REFERENCES

- [1] Bourbaki, Nicolas (1998), Elements of the History of Mathematics, Berlin, Heidelberg, and New York: Springer-Verlag, 301 pages, ISBN 3-540-64767-8.
- [2] Boyer, C. B.; Merzback (fwd. by Isaac Asimov), U. C. (1991), History of Mathematics, New York: John Wiley and Sons, 736 pages, ISBN 0-471-54397-7.
- [3] Bressoud, David (2002), Was Calculus Invented in India?, The College Mathematics Journal (Math. Assoc. Amer.) 33 (1): 2–13, doi:10.2307/1558972, JSTOR 1558972.
- [4] Bronkhorst, Johannes (2001), Panini and Euclid: Reflections on Indian Geometry, Journal of Indian Philosophy, (Springer Netherlands) 29 (1–2): 43–80, doi:10.1023/A:1017506118885.
- [5] Burnett, Charles (2006), The Semantics of Indian Numerals in Arabic, Greek and Latin, Journal of Indian Philosophy, (Springer-Netherlands) 34 (1–2): 15–30, doi:10.1007/s10781-005-8153-z.
- [6] Burton, David M. (1997), The History of Mathematics: An Introduction, The McGraw-Hill Companies, Inc., pp. 193–220.
- [7] Cooke, Roger (2005), The History of Mathematics: A Brief Course, New York: Wiley-Interscience, 632 pages, ISBN 0-471-44459-6.
- [8] Dani, S. G. (25 July 2003), Pythagorean Triples in the Sulvasutras, Current Science 85 (2): 219–224.
- [9] Datta, Bibhutibhusan (Dec 1931), Early Literary Evidence of the Use of the Zero in India, The American Mathematical Monthly 38 (10): 566–572, doi:10.2307/2301384, JSTOR 2301384.
- [10] Datta, Bibhutibhusan; Singh, Avadesh Narayan (1962), History of Hindu Mathematics : A source book, Bombay: Asia Publishing House.