

THEORETICAL AND EXPERIMENTAL STUDIES ON HYDRODYNAMICS OF A VERTICAL CONVERGING PNEUMATIC CONVEYING SYSTEM

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ABSTRACT

Pneumatic conveying is a practical form of transport that has been successfully applied in many facets of the chemical process industries. The present study deals with experimental studies on the hydrodynamic aspects using different types of solids like sago (Cycas revoluta), red lentil (Lens culinaris), black mustard (Sinapis nigra) and white mustard (Sinapis alba) in a short converging pneumatic conveying system incubated by various pressure taps throughout the transport tube for the study of the pressure drop profile at various length of the riser. A mathematical model was developed and the evaluated results obtained by theoretical analysis were compared with the experimental data. The pressure profile was also compared at different air and solid flow rates for different types of samples. The significance of converging was analyzed by making modification on the developed model so as to justify its advantage over vertical conveying system.

Keywords: *Conveying, Converging, Hydrodynamics, Transport tube, Pneumatic*

I INTRODUCTION

Pneumatic conveying is defined as the transport of particulate solids by a gaseous stream through any flow channel. Due to some technological advantage, this operation has been used in process industries for a long time, just for transport of several kinds of particulate solids, such as lime, coal, polymer pellets, soda ash and granular chemicals etc. However, the operation of pneumatic conveying is not limited to carrying particulate solids, but has been extended to other different processes such as preheating, drying, catalytic & other gas-solid reactions, as for example, drying particulate food materials & calcinations of minerals, the catalytic cracking of gas-oil and naphtha residues, suspension pre-heating as used in modern cement manufacturing process etc. Since the initiation of such technique for solid transportation, several researchers on its hydrodynamics and other transport phenomena of similar systems conducted numerous research works. Interest in pneumatic

transport dates back as early as 1924, *Cramp and Priestly* [1] being the pioneer investigators. They derived an expression for pressure drop by assuming the losses due to friction between the particles and the wall to be negligible. Horizontal transport of wheat was conducted and observed that at fluid velocities greater than twice the terminal settling velocity of a free falling wheat particle, the pressure drop due to solid was directly proportional to the solid loading ratio [2]. Specific pressure drop and the operating variables were interrelated in a manner such that the constants were expressed as a function of the square root of the dimensionless group. The pressure drop in vertical tube of small particles was analyzed and pointed out that for finely divided materials, the fluid flow relative to particle might change from turbulent in the initial stage of acceleration to steady region, thereby giving wide variations in the value of the drag coefficient. They concluded the pressure drop for flow was due to the sum of the pressure drop due to the carrier gas and solids [3]. Later it was considered that the pressure drop across a pneumatic transport line to be composed of static and frictional heads of the suspension as a whole [4]. The important conclusion obtained from research on vertical pneumatic transport is that the pressure drop due to solid alone may be derived by assuming that the particles behave as if they are freely suspended in the fluid stream and also the drag force was proportional to the square of the relative velocity between gas and the particles [5]. Later it was pointed out that the pressure drop could be estimated by summing up the contributions of the wall friction, particle friction, particle acceleration and static heads due to the particles and air [6]. Equations were derived for the additional pressure drop and particle velocity for in horizontal and vertical pipes, using materials such as glass beads, copper spheres, millet and gram seeds and was found that the velocity profile of particles in the vertical pipe was reported to be symmetrical and was not significantly affected by the particle size, density and mass flow ratio of air to the particles [7]. Based on literature survey at the Institute of Gas Technology, Chicago twenty important correlations for determining the pressure drop in vertical pneumatic conveying lines, was analyzed and presented in the "Coal Conversion systems Technical data book". The modified correlation (wherein pressure drop due to gas acceleration, particle acceleration, gas to pipe friction, solid to pipe friction, static heads of solids and that for the gas have been considered) was found to be quite simple and reasonably accurate. Investigations of the dependence of overall pressure drop on the superficial air velocity during pneumatic conveying in vertical pipes are reported [8]. The analysis confirmed that the friction pressure losses for the gas-solids suspension flow in vertical pipes can best be described using the correlational equation of *Kerker* [9]. Later it was observed that the total pressure drop consisted of three terms, namely acceleration of the particles, drop due to hold up of the particles and gas [10]. *Horio et al* suggested that the pressure drop in the riser, could be written as equal to the sum of (i) pressure drop for suspending solids, (ii) particle to wall frictional loss and (iii) acceleration loss [11]. Experimental study of vertical pneumatic conveying used a one-dimensional equation system and experimental techniques to provide a comprehensive description of vertical gas-solid two phase flow [12]. It was found that the frictional pressure drop was recognized to be an important component of the total pressure gradient in the riser [13]. At the same time three flow modes of Smooth transition from dilute to fluidized dense phase, Dilute-phase, unstable zone and slug flow and Dilute-phase only [14] were observed when bulk solid materials are transported in conventional pneumatic conveying systems. They are: (1) Smooth transition from dilute to fluidized dense

phase. (2) Dilute-phase, unstable zone and slug flow. (3) Dilute-phase only [14]. Studies on the effects of particle size and density on the fluid dynamic behavior of vertical gas-solid transport of group D particles on pneumatic conveying in dilute and dense phase flows were done [15]. Further studies showed that pressure drop in the acceleration region could be predicted using the uniform flow model if the proper value of the initial solid velocity was known. The equations for estimation of this value were proposed and the correlations for drag force co-efficient and friction factor that gave the most accurate results were selected [16]. Experimental investigation of inter particle collision rate in particulate flow used a high-speed camera and Particle Tracking Velocimetry (PTV) algorithms to focus on inter particle collisions and especially the collision rate [17]. Reports on the effect of Reynolds number, mass loading, and particle shape and size on pressure drop in a vertical gas-solids pneumatic conveying line were made and a state-of-the-art multiphase computational fluid dynamics (CFD) models were assessed by comparing their predictions to experimental data [18]. The conveying characteristics of various agricultural seeds like wheat, barley, sunflower and lentil were determined and in addition, the mechanical damage, germination rate, and vigour index of the seeds were also investigated experimentally. The physical properties like sphericity, densities, porosity, projection area and drag coefficient were determined [19]. Next year high speed video camera and pressure transmitters were used to study the dynamic behavior of the particles and its influence on pressure transmitters during conveying [20]. Two different types of plastic pellets were applied for the determination of distinguishing flow characteristics in dilute phase pneumatic conveying. The results show that the total pressure drop for polyolefin is higher than that for polystyrene. The solid friction factor decreases when the solid loading ratio is increased for both materials, but the friction factor for polyolefin decreases more with the solid-loading ratio than the polystyrene [21]. In the present investigation, the hydrodynamics of various material like *like sago (Cycas revoluta)*, *red lentil (Lens culinaris)*, *black mustard (Sinapis nigra)* and *white mustard (Sinapis alba)* were elaborately studied by calculating the pressure drop profile along the height of the riser. Further the experimental analysis was compared with the theoretical data obtained from the developed model. The effect of converging of the vertical tube was also thoroughly studied so as to justify its application range.

II MATERIALS AND METHODS

A basic study was done by considering various materials like Red Lentil (*Lens culinaris*) which is a grain legume widely grown in Turkey for food being an important source of protein, Sago (*Cycas revoluta*) which is a starch extracted from the pitch of sago palm stems, Black mustard (*Sinapis nigra*) which is an annual weedy plant cultivated for its seeds, commonly used as spices, White mustard (*Sinapis alba*) which are hard round seeds containing oily embryo free from starch with a color ranging from yellow to light brown, to see the effect of size, density on the pneumatic conveying characteristics. The flow rate of each material was varied to have an analysis of the pressure drop along the riser height. The typical characteristics of the materials are listed in Table 1.

TABLE 1
Characteristics of Materials used

Material	Average Diameter (mm)	Density (kg/m³)	Shape
Small Sago	1.61	1240	Spherical
Large Sago	3.75	1170	Spherical
Red Lentil	2.75	1504	Convex lens
Black Mustard	1.905	1027.5	Spherical
White Mustard	1.85	1570	Spherical

III EXPERIMENTAL ANALYSIS

The existing experimental set-up (Figure 1) consist of a transport tube having a length of 2.7m and diameter converging from 0.0762m to 0.0508m with all other accessories for gas and solid feeding and solid cyclone separation. Before beginning the experiment we estimate all the physical parameters of the solid particles used for transportation. The storage bin is first loaded with desired particulate solids. Air is sent from the blower and the solid in the bin is charged slowly by opening the valve (V_2) fitted at the bottom of the storage bin so that particles can be fluidized continuously without retaining those particles at the bottom of the converging tube. After completing the charging V_2 is closed. For easy feeding of the solid and to eliminate the leakage of air through the solid feeding line compressed air is sent through the tube fitted co-axially with the solid feeding line. This airflow is adjusted so as to restrict the air leakage from the transport tube towards the solid feeding line. The solid particles observed to be transported by the air flowing through the vertical converging transport tube & separated from the air stream by the cyclone separator fitted at top of the vertical transport tube. The solid feeding rate is verified by closing intermittently valve (V_1) fitted with the leg of the cyclone separator. Differential pressure is measured across the various pressure taps and the base pressure taps (bottom most) of the transport tube. The data thus collected for various differential pressures at different height of the transport tube were plotted against the height of the transport tube with different solid particles at constant airflow rate & compared with the theoretical differential pressure profiles.

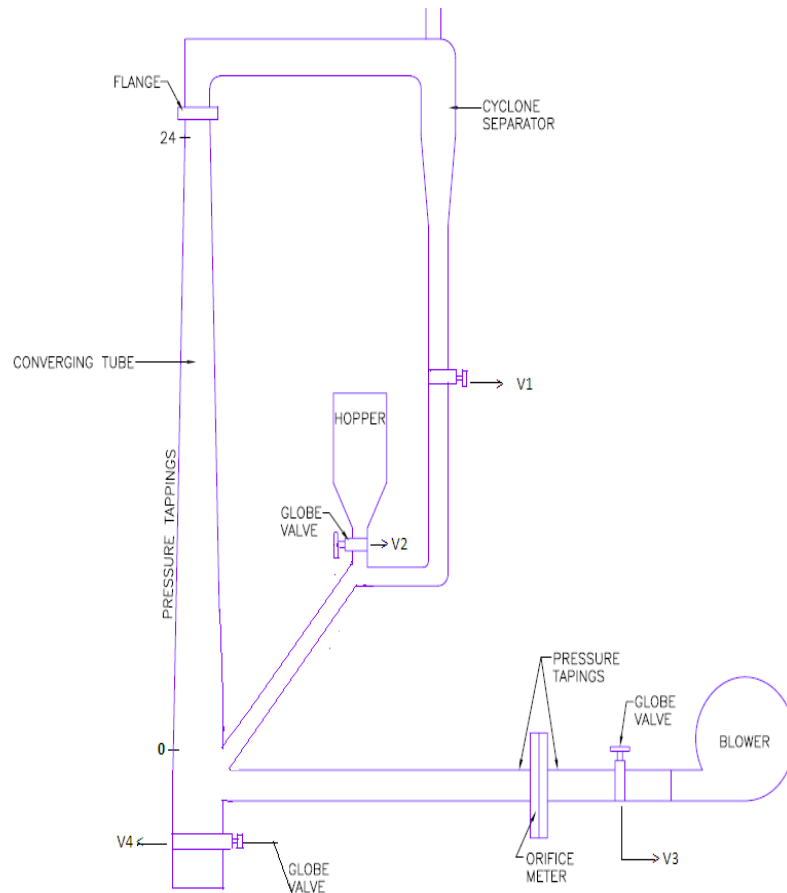


Fig. 1: Short Converging Vertical Pneumatic Conveying system

IV MATHEMATICAL MODEL

The mathematical model on dynamics of the particulate solids in the vertical pneumatic conveying system through uniform transport tube [12] has been modified considering the same assumption in the existing computational model to accommodate the converging tube geometry for theoretical prediction of pressure drop profile, for dilute phase pneumatic transport through converging riser tube.

The frictional force of the particle is estimated by the *Konno* [22] ratio relationship as given below

$$F_s = \frac{0.75 \rho_g C_d}{d_p} (u_g - u_p)^2 \left(\frac{\pi d_t^2}{4} \right) dh (1 - \varepsilon) \quad (1)$$

The buoyant force due to the fluid medium is negligible in comparison with the gravitational force acting on the particle. Where ρ_g = Density of the gas, ε = Initial porosity, u_g = Gas velocity, u_t = Terminal velocity, u_p = Particle velocity, d_t = Total diameter, C_d = Drag co-efficient

d_t = tube diameter at any arbitrary height, h of the converging tube is given by

$$d_t = \frac{[(H-h)d_1 + h \times d_2]}{H} \tag{2}$$

H = axial length of the converging tube (or, vertical height); d_1 = bottom most diameter of the tube;

d_2 = top most diameter of the tube;

h =any arbitrary height considered from the bottom most point of the converging tube

A generalized expression for drag co-efficient has been suggested by *Morsi and Alexander* [23] follows:

$$C_d = \left(\frac{k_1}{Re_p} \right) + \left(\frac{k_2}{Re_p^2} \right) + k_3 \tag{3}$$

Where, $Re_p = \frac{(u_g - u_p)d_p \rho_g}{\mu_g}$

and k_1, k_2, k_3 are empirical constants assuming different values for different range of Re_p as shown in Table 2

TABLE 2

Values of empirical constants for different ranges of Re_p

For	k_1	k_2	k_3
$Re_p \leq 0.1$	24.0	0.0	0.0
$0.1 < Re_p \leq 1.0$	22.73	0.0903	3.69
$1 < Re_p \leq 10$	29.166	-3.8889	1.222
$10 < Re_p \leq 100$	46.5	-116.67	0.6167
$100 < Re_p \leq 1000$	98.33	-2778.0	0.3644
$1000 < Re_p \leq 5000$	148.62	-47500.0	0.357
$5000 < Re_p \leq 10000$	490.546	-578700.0	0.46
$Re_p > 10000$	1662.5	-5416700.0	0.5191

4.1 Pressure drop Calculation

The pressure drop across the differential height, dh may be considered as the summation of five different pressure losses.

$$dp_t = dp_{sg} + dp_{fg} + dp_{ss} + dp_{fs} + dp_{as} + dp_{ag} \quad (4)$$

Where, dp_t = total pressure drop across the element

The pressure losses on the right hand side of equation (4) may be expressed in terms of dh ,

Pressure drop due to static column of fluid,

$$dp_{sg} = \rho_g \varepsilon g dh \quad (5)$$

Pressure drop for the fluid friction with the wall,

$$dp_{fg} = \frac{2f_g \rho_g u_g^2 \varepsilon}{d_t} dh \quad (6)$$

Pressure drop due to static column of solid particles,

$$dp_{ss} = \rho_p (1 - \varepsilon) g dh \quad (7)$$

Pressure drop as a result of solid friction with the tube wall,

$$dp_{fs} = \frac{2f_s \rho_p (1 - \varepsilon) u_p^2}{d_t} dh \quad (8)$$

Pressure drop due to acceleration of the solid particles,

$$dp_{as} = \rho_p (1 - \varepsilon) u_p^2 \quad (9)$$

And pressure drop due to acceleration of the gas to the carrying velocity,

$$dp_{ag} = \frac{1}{2} \rho_g \varepsilon u_g^2 \quad (10)$$

$$\text{Now, } Re_g \text{ gas Reynolds number} = \frac{u_g \rho_g d_t}{\mu_g}$$

Since, Re_g ranges between 20000 and 60000 in the present set of experiments, using standard friction factor versus Reynolds number plot. An empirical correlation may be developed for smooth pipe as follows:

$$f_g = 0.08 (Re_g)^{-0.25} \\ = 0.08 \left(\frac{u_g \rho_g d_t}{\mu_g} \right)^{-0.25} \varepsilon^{0.25} \quad (11)$$

For vertical transport, at the low solid to air ratios (less than 5 to 10 by weight) normally employed in dilute

phase pneumatic, it may be assumed that

$$u_g \cong (u_g)_{\text{sup}}$$

$$\text{Where, } u_g = \text{actual gas velocity} = \frac{m_g}{\rho_g (\pi d_t^2 / 4)} \quad (12)$$

where m_g = kg of air flow per second

$$(u_g)_{\text{sup}} = \text{superficial gas velocity}$$

And $(u_g - u_p) \cong u_t$ for vertical transport at low solid ratio

Where u_t = terminal free settling velocity of the particles in air and f_s = Friction factor

For vertical transport of spherical particles as the flow zone is not likely to be laminar always, Newton's law may be used to estimate the free settling velocity of the particles which is given by

$$u_t = \left[\frac{4}{3} \left(\frac{\rho_s - \rho_g}{\rho_g} \right) \frac{g d_p}{C_d} \right]^{0.5}, \quad f_s = \left(\frac{3}{8} \right) \left(\frac{\rho_g C_d d_t}{d_p \rho_p} \right) \left(\frac{u_g - u_p}{u_p} \right)^2$$

V RESULTS AND DISCUSSIONS

The experimental pressure drop data collected from pressure taps at different height of the riser using different particulate solids are given in the table

5.1. Experimental Pressure Drop calculation of various materials

Figure 2 shows the experimental pressure drop profile for small sago, large sago, white mustard, black mustard, red lentil and air with respect to the solid feed point. The corresponding plot mentioned below show that with riser height the slope of the curves gradually increases and continue to increase till the end of the riser. Thus it is observed that the pressure profile increases exponentially. The exponential nature is observed for both cases (with or without solid) and comparative plots are plotted. This phenomenon implicates that the particle cannot reach the steady state velocity at any stage of the riser. This is due to the fact that in case of converging tube the gas velocity is not constant throughout the height which is observed in case of converging tube, rather the gas velocity is continuously increasing due to the decrease in cross-sectional area along the riser height in case of converging riser tube. So the final steady state velocity to be attained by the transporting solid particles is not set

at a constant value rather continuously increasing along the riser height. In other words we may comment that the particles are under continuous accelerating force along the riser tube.

It is seen that the smallest diameter particle that is small sago is occupying higher position than others. Then comes white mustard having a diameter of 1.85mm, then comes black mustard (dia=1.905mm), then red lentil(dia=2.75mm) and lastly large sago with a diameter of 3.75mm. This shows that the pressure drop profile decreases with increasing diameter of the particle. This can be well justified by the fact that with increasing diameter, the running particles are becoming more sluggish during their transportation through the transport tube.

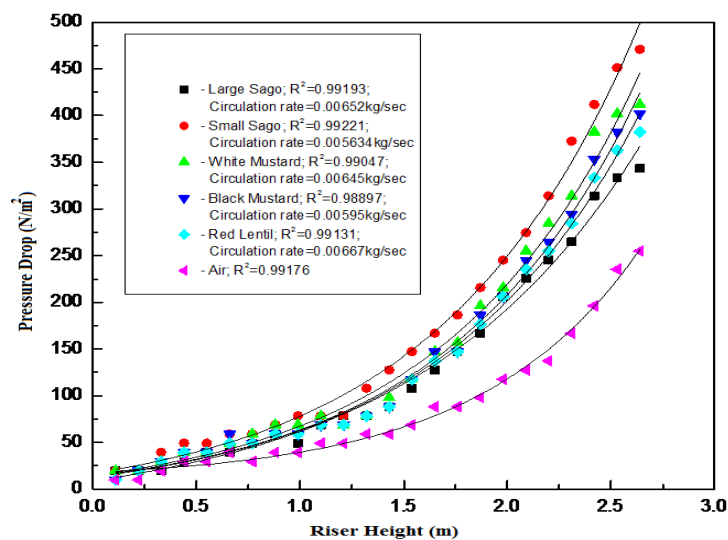


Fig. 2: Experimental Pressure Drop for all the materials and air

5.2. Comparative study between experimental and theoretical pressure drop

A comparative experimental and theoretical pressure drop profile for small sago(dia=1.61mm), white mustard(dia=1.85mm), black mustard(dia=1.905mm), red lentil(dia=2.75mm) and large sago(dia=3.75mm) is represented in Figure 3. These plots show variation of pressure drop profiles with different diameters and densities of the particulate solids. Theoretical plots are derived from the model by using computer program. In all these figures there is a good match between the experimental and theoretical results with certain discrepancies which may be due to the unequal distribution of the particles in the radial direction of the tube. This aspect differs from the basic assumption of the theoretical model that the distribution of particles are uniform in the radial direction. This discrepancy is seemed to increase with the particles size which may be due to the fact that the larger particles are statically charged during the friction between the tube wall and the particles, as a result of which the particles are not accelerated properly due to the static forces in opposite

direction to their transport. And hence the experimental pressure drop profile lies below the theoretical pressure drop profile.

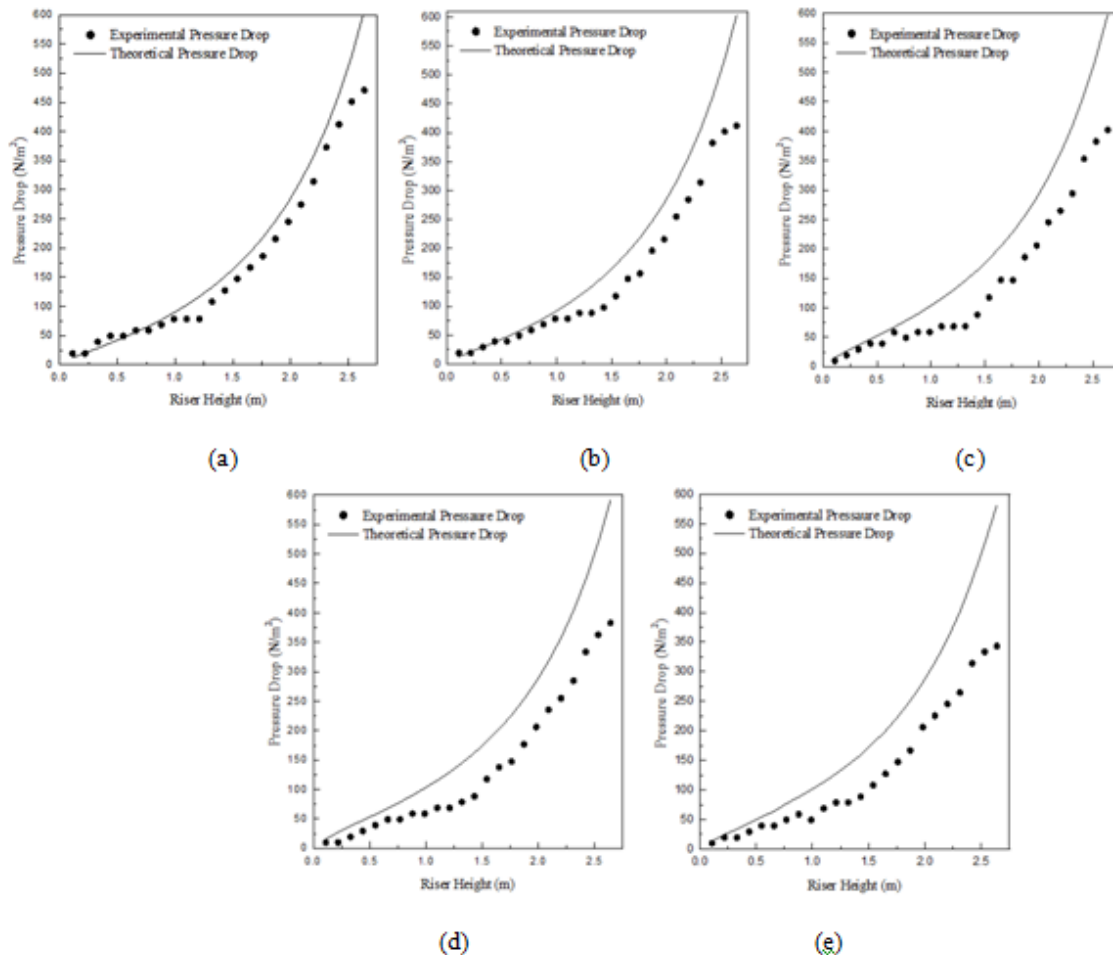


Fig. 3: Comparative study between experimental and theoretical pressure drop of (a) Small sago (b) White mustard (c) Black mustard (d) Red lentil (e) Large sago

5.3. Comparative study of pressure drop profile for two different circulation rates

Figure 4 shows comparative pressure drop profile at different circulation rates for small sago(circulation rate of 0.005634kg/sec and 0.019kg/sec), large sago(circulation rate of 0.00652kg/sec and 0.0186kg/sec), white mustard(circulation rate of 0.00645kg/sec and 0.021kg/sec), black mustard(circulation rate of 0.00635kg/sec and 0.022kg/sec) and red lentil(circulation rate of 0.00667kg/sec and 0.019kg/sec) respectively. It is observed that with higher solid circulation rate the exponential trend is more. Due to the higher amount of accelerating solid particles at higher solid circulation rate at any height of the riser the total pressure drop is high for high solid circulation rate.

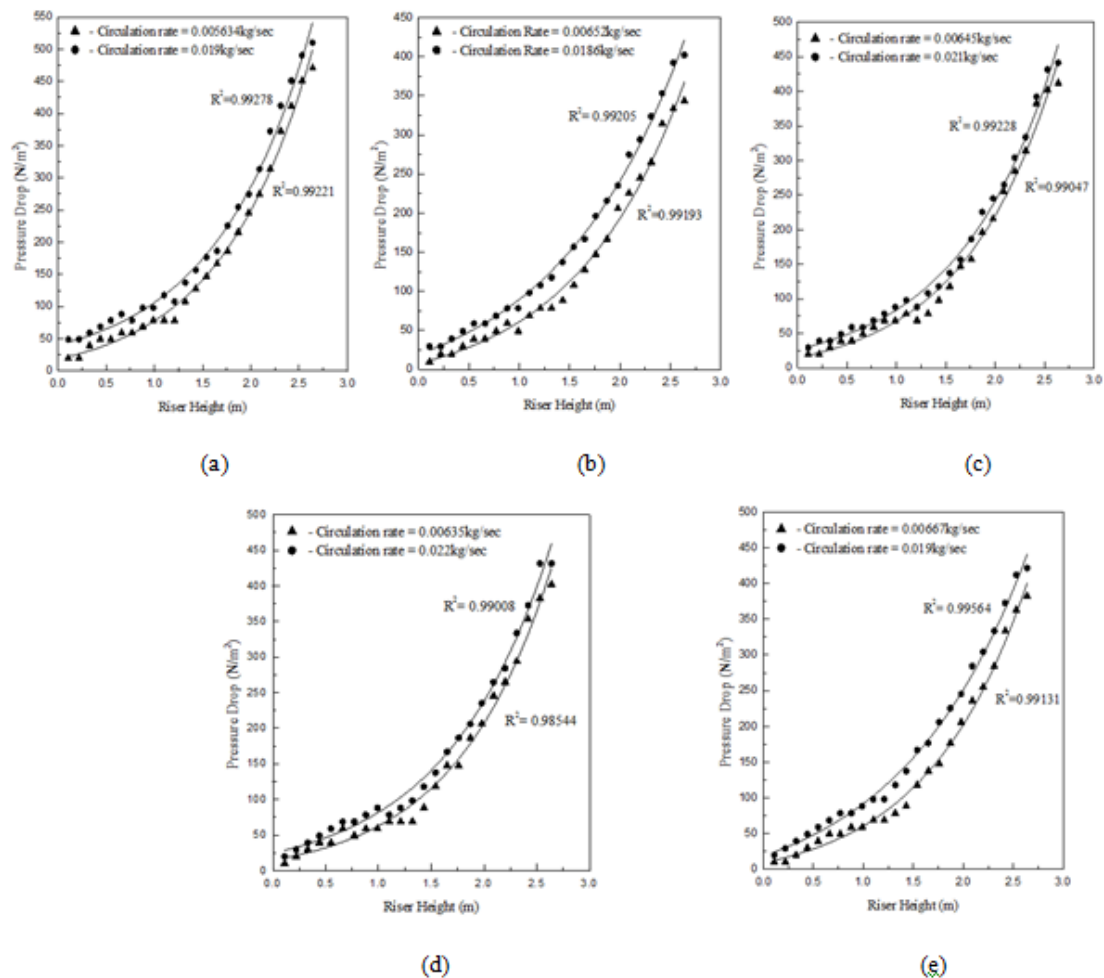


Fig. 4: Comparative study of pressure drop profile for two different circulation rates (a) Small sago (b) Large sago (c) White mustard (d) Black mustard (e) Red lentil

5.4. Comparative study of pressure drop profile for converging and uniform tube

Figure 5 shows comparative theoretical pressure drop profiles between the uniform and the converging riser tube. These theoretical plots are derived from the model by using computer program. In this figures the profiles show that the pressure drop of converging tube is gradually increasing at a faster rate than that of the uniform tube. The gradient of pressure drop profile is gradually increasing with the riser height which doesnot happen in case of uniform tube. This is because of the fact that the particulate solids in case of converging tube is under constant acceleration and are transported subsequently with increasing velocity and never attaining the steady state velocity.

In all the plots described above, clear deviations from the actual plots are seen for a few points at the end of the curve. This deviation may be due to the fact that at the top section of the transport tube, the flow of air is more

turbulent just at the entry of the cyclone. Subsequently the motion of the accelerated particles are also hindered due to the same reason. The combined effect may be the cause of the stray points at the end of the curve.

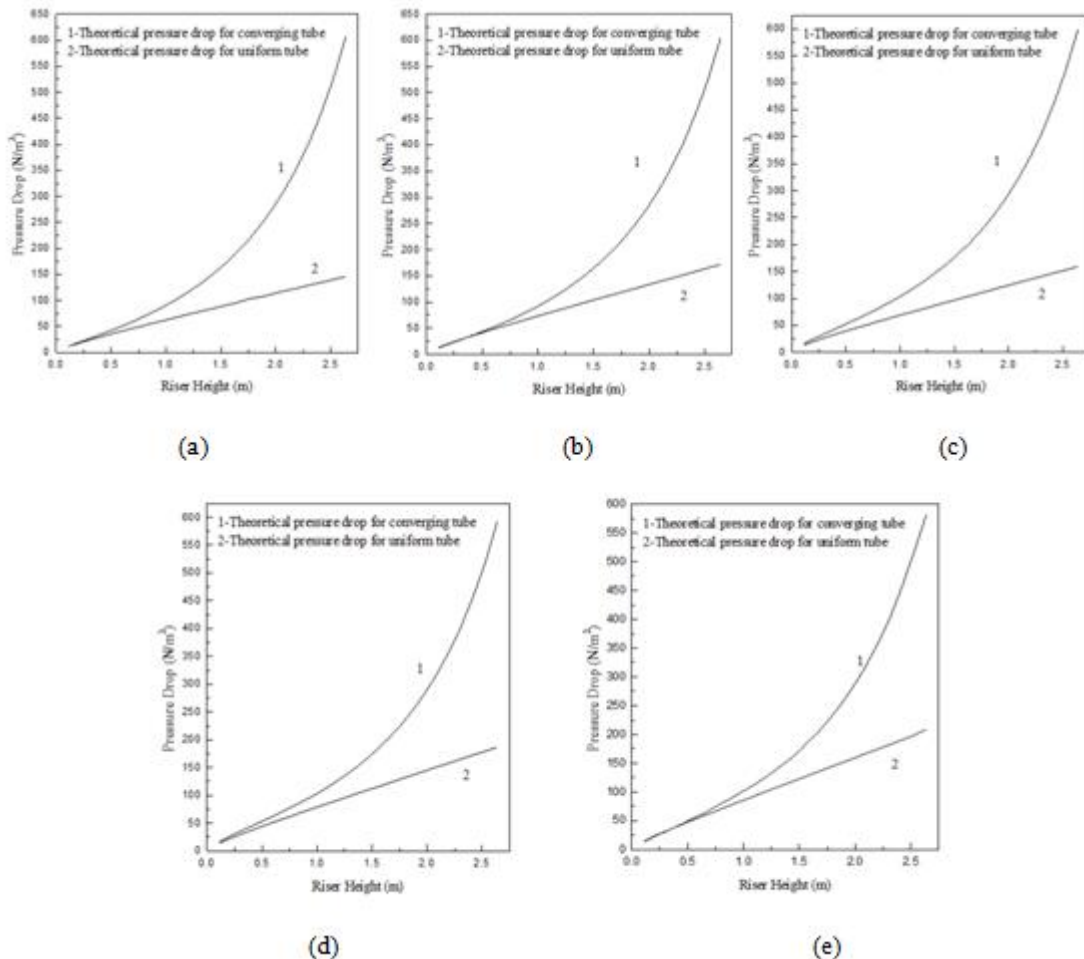


Fig. 5: Comparative study of pressure drop profile for converging and uniform tube (a) Small sago (b) White mustard (c) Black mustard (d) Red lentil (e) Large sago

VI CONCLUSION

The results of pneumatic conveying characteristics of sago, red lentil, black mustard and white mustard was estimated and it was determined that the pressure drop of each kind of particles is increasing along the riser height till the end of the riser in case of converging tube. This is due to the fact that in case of converging tube the gas velocity is not constant throughout the height, rather the gas velocity is continuously increasing due to the decrease in cross-sectional area along the riser height in case of converging riser tube. So the final steady state velocity to be attained by the transporting solid particles is not set at a constant value rather continuously increasing along the riser height. In other words we may comment that the particles are under continuous accelerating force along the riser tube. The highest pressure drop was obtained for small sago(dia=1.61mm) whereas the lowest pressure drop was obtained for large sago(dia=3.75mm). It is also observed that the pressure

drop increases as the solid flow rate or the circulation rate of the solid increases. It may also be noted that the pressure drop of converging tube is gradually increasing at a faster rate than that of the uniform tube. The gradient of pressure drop profile is gradually increasing with the riser height which doesnot happen in case of uniform tube. This is because of the fact that the particulate solids in case of converging tube is under constant acceleration and are transported subsequently with increasing velocity and never attaining the steady state velocity. Therefore it may be concluded that the transfer coefficients are much larger in case of converging tube than in case of uniform one. Since the particles Reynolds number is continuously increasing throughout the riser tube in case of converging geometry. So, on the basis of the all specific points it can be well suggested that in any operation involving vertical riser a converging transport tube is more efficient.

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