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Effect of Aspect Ratio and Damping Factor on Non-linear Steady State Forced Vibration of Rectangular Isotropic Plates

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Abstract. The linear and non-linear steady state forced vibration analysis of rectangular plates has been carried out to investigate the influence of aspect ratio and damping coefficient. The finite element analysis has been done based on the kinematics of first order shear deformation theory. A C0 continuous, eight-noded serendipity quadrilateral shear flexible element with five degrees of freedom has been used for the analysis. The geometric non-linearity is included in the analysis using von Kármán's assumption for small strains and moderately large deflection. The governing equation of motion has been obtained in time domain and the entire non-linear steady state frequency response curve consisting of stable and unstable regimes has been obtained employing shooting technique along with arc length and pseudo-arc length continuation schemes. The frequency response curves, temporal response and phase plane plots have been obtained to demonstrate the peculiar nature of the nonlinearity.

Keywords: Forced vibration analysis, Steady state response, Non-linear analysis, shooting method.

1. Introduction

Thin plate structures are used in a variety of fields, including automotive and building engineering, as well as space technology. Due to severe industrial standards, demand for this form of construction has exploded, particularly in aviation vehicles where low weight is critical. When exposed to dynamics load these thin structural components may undergo large deformation, so incorporation of geometric nonlinearity becomes imperative while designing these type of structures.

The free vibration responses analysis of anti-symmetric laminated angle ply plates based on FEM by Reddy [1979]. The six nodded triangular shell element is used in the study based on higher order FEM by Yu [1994]. Vuksanovic [2000] using single layer discrete models to investigate static, dynamic, free vibration and buckling behavior of laminated plate based on finite element method. Aagaah et al. [2003] studied bending behaviour of multilayered rectangular composite plate using finite element method. Setoodeh and Karami [2004] proposed 3D layer-wise finite element models to analysed static, free vibration and buckling behavior of thick anisotropic laminated composite plates under different supports. Reddy [2004] explains the number of examples of static, dynamic, stability and vibration of laminated structures for different boundary conditions, geometry and materials. Ferreira et al. [2005] used radial basis functions combined with first-order deformation theory to investigate the symmetrically laminated composites. Subramanian [2006] using higher order theories combined with finite element approach to analysed dynamic behaviour of laminated composite beams. Bending analysis of isotropic rectangular plate on different boundary conditions and loads by Vanam et al. [2012] based on FEM. The non-linear steady state response of isotropic rectangular plate investigated for different boundary conditions based on FEM by Beg et al. [2018].

Direct time integration approaches are computationally more intensive and unable to predict unstable branches as we have seen by the work of Patel et al. [2006] and Ibrahim et al. [2008]. The number of equations is independent of the number of harmonics, unlike frequency domain methods. Ibrahim et al. [2009] developed a modified shooting technique for forecasting non-linear system's periodic responses directly from the solution of second-order equations.

The effect of aspect ratio and damping factor on the nonlinear steady state forced vibration response has not been properly examined, based on open literature. A uniformly distributed transverse harmonic force is applied to the rectangular plate. For following the complete non-linear frequency response curve, the direct integration technique is insufficient. The modified shooting method given here is utilized to capture the entire behavior of a nonlinear frequency response without making any assumptions about the contributing modes. Frequency response curves, steady state response history, and phase-plane plots were used to explore the effects of aspect ratio and damping factor on non-linear frequency response.

2. Finite Element Formulation

The Fig. 1 shows the geometry and coordinate system of a rectangular isotropic plate subjected to an external forcing function.

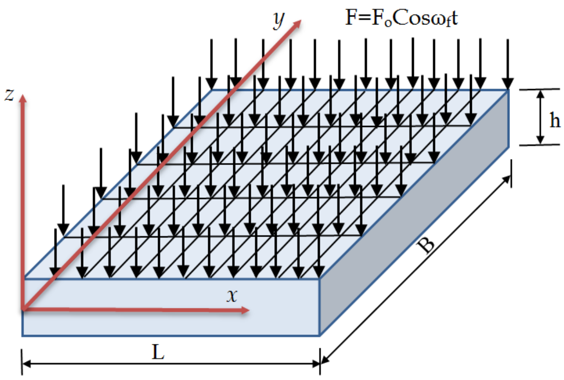

Fig. 1 Geometry and coordinates of rectangular isotropic plate

A C^0 continuous, 8-noded serendipity shear flexible quadrilateral element having 5 degrees of freedom has been used

$$\{u\}^e = \{u_0\ v_0\ w_0\ \phi_x\ \phi_y\}^T \tag{1}$$

The used deformation field is:

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,z,t) + z\phi_x(x,y,t) \\ v(x,y,z,t) &= v_0(x,y,z,t) + z\phi_y(x,y,t) \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{2}$$

Where, $(u_o, v_o, w_o, \phi_x, \phi_y)$ are unknown parameter to be determined. (u_0, v_0, w_0) denote the mid-plane displacements at $z = 0$.

Using von Karman's assumption for small strains and moderately large rotation, strain field can be written in terms of mid-plane displacement as:

$$\{\varepsilon\} = \{\varepsilon_{xx}\quad \varepsilon_{yy}\quad \gamma_{xy}\quad \gamma_{xz}\quad \gamma_{yz}\}^T = \begin{Bmatrix} \varepsilon_p^L \\ 0 \end{Bmatrix} + \begin{Bmatrix} z\varepsilon_b \\ \varepsilon_s \end{Bmatrix} + \begin{Bmatrix} \varepsilon_p^{NL} \\ 0 \end{Bmatrix} \tag{3}$$

Where the membrane strain ε_P^L , bending strain ε_b , transverse shear strains ε_s and the non-linear membrane strain ε_P^{NL} vectors are defined as

$$\{\varepsilon_P^L\} = \left\{\frac{\partial u_0}{\partial x} \quad \frac{\partial v_0}{\partial y} \quad \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}\right\}^T, \qquad \{\varepsilon_b\} = \left\{\frac{\partial \phi_x}{\partial x} \quad \frac{\partial \phi_y}{\partial y} \quad \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x}\right\}^T$$
$$\{\varepsilon_S\} = \left\{\phi_x + \frac{\partial w_0}{\partial x} \quad \phi_y + \frac{\partial w_0}{\partial y}\right\}^T, \ \{\varepsilon_P^{NL}\} = \left\{\frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2 \quad \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 \quad \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}\right\}^T \tag{4}$$

The membrane stress resultant $\bar{N} = \{N_{xx} \quad N_{yy} \quad N_{xy}\}^T$, moment resultant $\bar{M} = \{M_{xx} \quad M \quad M_{xy}\}^T$ and transverse shear stress resultant $\bar{Q} = \{Q_{xz} \quad Q_{yz}\}^T$ vectors are related to the membrane $\varepsilon_p = \varepsilon_P^L + \varepsilon_P^{NL}$, bending ε_b and transverse shear ε_s strain vectors through the constitutive relation as.

$$\begin{Bmatrix}\bar{N}\\ \bar{M}\end{Bmatrix} = \begin{bmatrix}A & B\\ B & D\end{bmatrix}\begin{Bmatrix}\varepsilon_p\\ \varepsilon_b\end{Bmatrix}, \quad \bar{Q} = Y\varepsilon_s \tag{5}$$

Kinetic energy of the plate is given by equation (4), where ρis the mass density

$$T = \frac{1}{2}\iint \left(\rho h(\dot{u}_0^2 + \dot{v}_0^2 + \dot{w}_0^2) + \frac{\rho h^3}{12}(\dot{\phi}_x^2 + \dot{\phi}_y^2)\right) dxdy \tag{6}$$

The total potential energy functional V is given by:

$$V = \frac{1}{2}\iint \left(d^T K d + \frac{1}{3} d^T K_1 d + \frac{1}{6} d^T K_2 d\right) dxdy - \iint F w_0 dxdy \tag{7}$$

The governing equation of motion can be expressed as using the standard FE assembly process and taking into account dissipative forces as:

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [\mathbf{K} + (1/2)\mathbf{K_1}(U) + (1/3)\mathbf{K_2}(U)]\{U\} = \{F\} \tag{8}$$

Where [**K**] is the linear stiffness matrix, [$\mathbf{K_1}$] and [$\mathbf{K_2}$] are non-linear stiffness matrices linearly and quadratically based on the field variables respectively.

Using Rayleigh proportional damping model, the damping matrix [C] is taken as:

$$[C] = \hat{\alpha}[M] + \beta[K] \tag{9}$$

Where, $\beta = \frac{\xi}{\omega_n}$; $\hat{\alpha} = \xi\omega_n$ (ω_n: natural frequency, ξ: modal damping factor;).

The modified shooting method is used for solving governing equations of motion [8]. For tracing the unstable region of steady state response curves, the arc length approach is used. At the sharp turning points, solution is not converged with shooting method then arc length method is used in continuation for find the corresponding converged frequency near sharp turns.

3. Results and Discussion

The steady state periodic response curve of isotropic plates for geometrically linear (GL) as well as for geometrically non-linear (GNL) case has been studied. The effect of aspect ratio and damping

factors on the GL and GNL steady state forced vibration response is analysed. The first fundamental mode is focused in this study and harmonic force $F=F_0\cos\omega t$ is uniformly distributed in the transverse direction over the entire surface of the plate.
The material properties used in this analysis are:
Young modulus, E = 71.7 GPa, density "ρ" = 2740 kg/m^3, Poisson's ratio, ν= 0.33.

The different boundary conditions used in this analysis are:
Clamped with immovable edge (**CCCC**) are:

$$v = u = w = \varphi_1 = \varphi_2 = 0; \qquad \text{at } x = 0,\ L \text{ and } y = 0,\ B$$

Simply supported with movable edge (**SSSS**) are:

$$v = w = \varphi_2 = 0; \qquad \text{at } x = 0, L$$

$$u = w = \varphi_1 = 0; \qquad \text{at } y = 0,\ B$$

3.1 Effect of aspect ratio (CCCC, L/B = 100, F_0 = 5 kPa, ξ = 0.010)

The Fig. 2 shows the effect of aspect ratio on the GL and GNL frequency response curves of rectangular isotropic plates. The frequency response graphs show that when the aspect ratio (L/B) increases, the peak amplitude of both the GL and GNL responses increases. The response amplitude increases proportionally with the aspect ratio for linear analysis, whereas the increase in response amplitude with the increase in aspect ratio is substantially reduced for geometric non-linearity. The peak non-dimensional amplitude in linear analysis is much larger as compared to the nonlinear analysis, with peak amplitude in linear analysis being 1.12, 3.51, and 4.79 times that of nonlinear analysis for L/B = 0.5, 1 and 2, respectively.
The steady state response history for the linear as well as non-linear analysis reveals equal positive and negative half cycle amplitude with equal time in tension and compression. The asymmetric nature of the phase plane plots with the inclusion of non-linearity depicts significantly large higher harmonic contribution unlike linear analysis where the phase plane plot reveal only dominant fundamental harmonic.

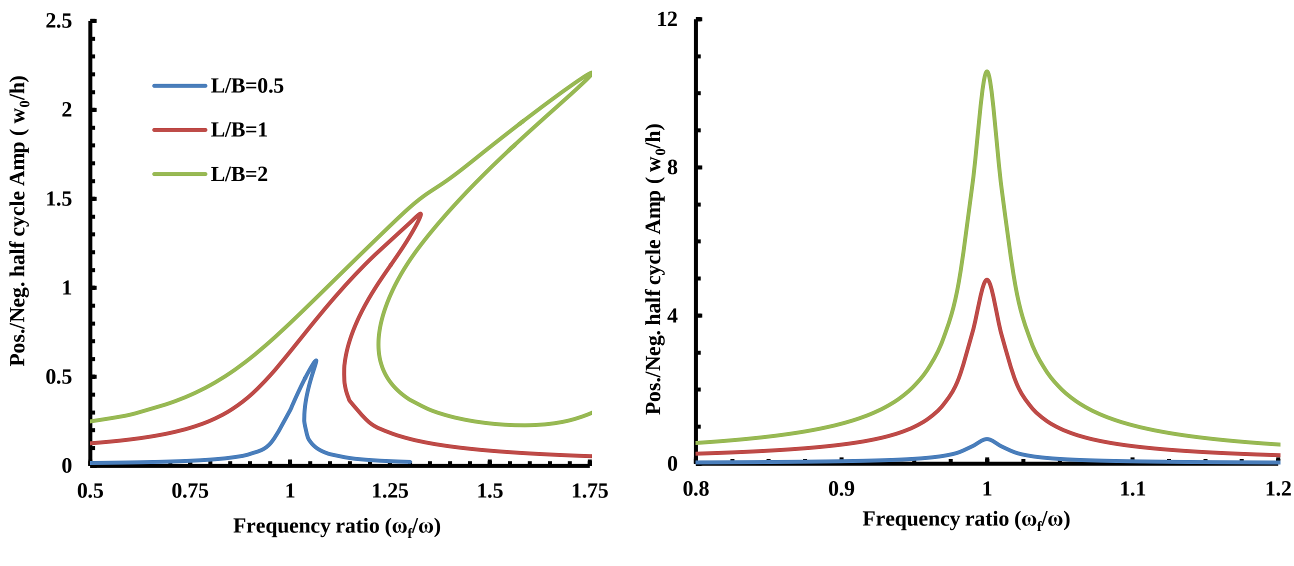

Fig. 2 Steady state frequency response curves, (i) geometrically non-linear (GNL), (ii) geometrically linear

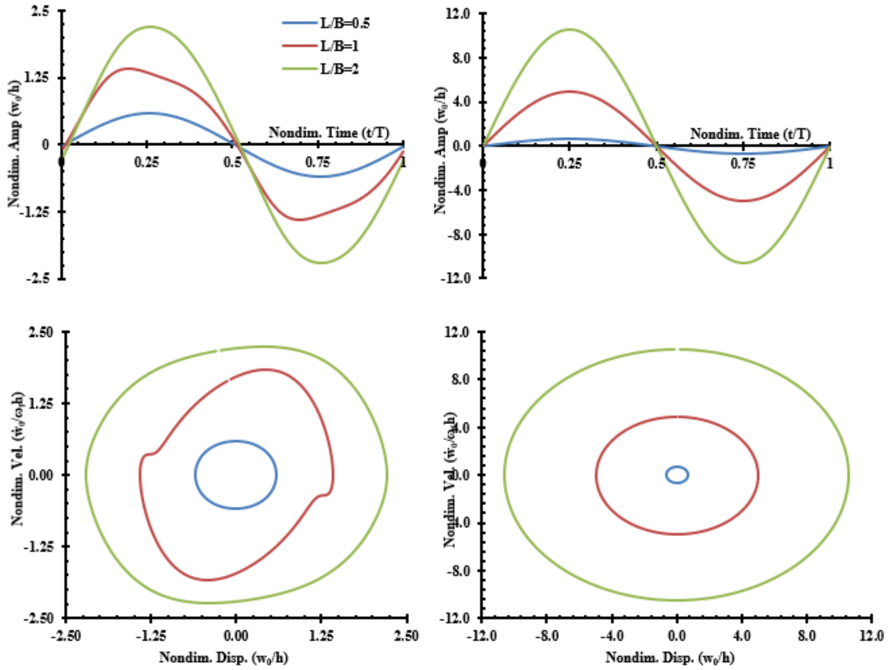

Fig. 3 Steady state response history and phase plane plots

3.2 Effect of damping factor (SSSS, L/B = 1; B/h = 100, F_0= 5 kPa)

The effect of the damping factor on the forced vibration response curves has been depicted in the figure Fig. 4. It is clear from these graphs that with the increase in damping factor the peak amplitude decreases in GL as well as in GNL analysis. The peak non-dimensional amplitude in the GL analysis being 5.37, 3.05 and 2.27 times the peak amplitude obtained from GNL analysis for ξ = 0.005, 0.010 and 0.015 respectively.

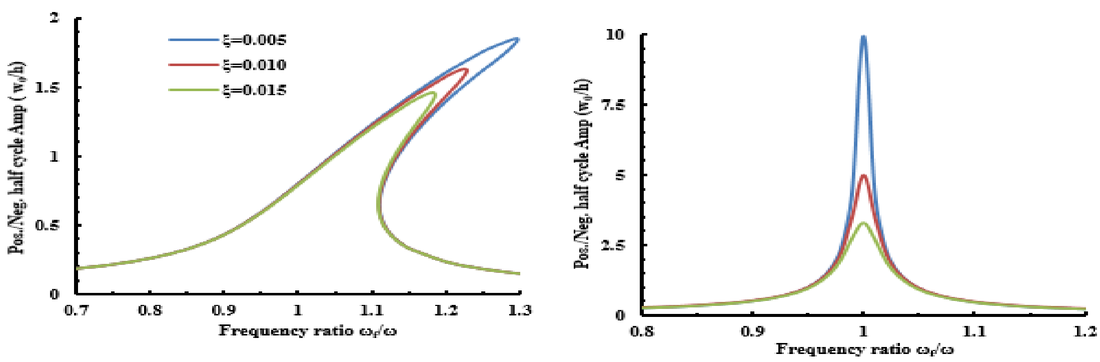

Fig. 4 Steady state frequency response curves, (i) geometrically non-linear (GNL), (ii) geometrically linear (GL)

4. Conclusion

The influence of aspect ratio and damping factor on the geometrically linear (GL) and geometrically non-linear (GNL) forced vibration responses of plates has been investigated. The frequency response graphs indicate that the peak amplitude of both the GL and GNL responses increases as the aspect ratio (L/B) increases. In both the GL and GNL studies, the peak amplitude drops when the damping factor is increased. The proposed method is computationally efficient since it does not require pre-assumptions of participating modes and traces the whole response curve, including stable and unstable branches, in a small number of iterations.

References

1. Reddy J.N., Free vibration of anti-symmetric, angle ply laminated plates including transverse shear deformation by the finite element method, Journal of Sound and Vibration, 66(1979): pp.565-576.
2. Yu, H., 1994. “Higher-order finite element for analysis of composite laminated structures” Compos. Struct. , vol.28, pp.375–383.
3. Vuksanovic, Dj., 2000. “Linear analysis of laminated composite plates using single layer higher-order discrete models” Composite Structures,vol. 48,pp. 205-211.
4. Aagaah, M., Mahinfalah, M., and Jazar, G., 2003. “Linear static analysis and finite element modeling for laminated composite plates using third order shear deformation theory” Composite Structures, vol. 62,pp. 27–39.
5. Setoodeh, A., and Karami, G., 2004. “Static, free vibration and buckling analysis of anisotropic thick laminated composite plates on distributed and point elastic supports using a 3-D layer-wise FEM” Engineering Structures,vol. 26, pp.211–220.
6. Reddy, J., 2004. “Mechanics of Laminated Composite Plates and Shells” Theory and Analysis CRC Press, Boca Raton, FL.
7. Ferreira, A.J.M., Roque, C.M.C. and Jorge, R.M.N., 2005. “Free vibration analysis of symmetric laminated composite plates by FSDT and radial basis functions”, Computer Methods in Applied Mechanics and Engineering, 194, pp. 4265–4278.
8. Subramanian, P., 2006. “Dynamic analysis of laminated composite beams using higher order theories and finite elements”, Composite Structures, 73, pp. 342–353.
9. Vanam B.C.L., Rajyalakshmi M. and Inala R., Static analysis of an isotropic rectangular plate using finite element analysis (FEA), Journal of Engineering Mechanical Research, 4 (2012): pp. 148-162.
10. Beg, M.S., Khan, A.H. and Yasin, M.Y., 2018, June. Effect of boundary conditions on the non-linear forced vibration response of isotropic plates. In IOP Conference Series: Materials Science and Engineering , IOP Publishing. 377 (1) p. 012089. DOI: 10.1088/1757-899X/377/1/012089.
11. Patel B P, Singh S, Nath Y 2006 Stability and nonlinear dynamic behaviour of cross-ply laminated heated cylindrical shells. Latin American Journal of Solids and Structures 3: 245–261.
12. Ibrahim SM, Patel B P, Nath Y 2008 On the nonlinear vibrations of clamped oval shells. Proceedings of the 9th Biennial ASME Conference on Engineering Systems Design and Analysis (ESDA 2008), July 7–9, 2008, Haifa, Israel.
13. Ibrahim SM, Patel B P, Nath Y 2009 Modified shooting approach to the non-linear periodic forced response of isotropic/composite curved beams, International Journal of Non-Linear Mechanics 44:1073-1084.