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A NOTE ON THE FLUID FLOW IN A CURVED PIPE OF ELLIPTICAL CROSS SECTION

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ABSTRACT

Srivastava has considered the motion of fluid in a curved pipe of elliptical cross section. In the present note some numerical work has been carried out using the results of Srivastava. The results obtained demonstrate its utility to flow in cardiovascular system and pulmonary blood vessels.

Keyword: Curved Pipe Flow, Carotid Artery, Ellipticity, Flux Ratio

I. INTRODUCTION

Several attempts have been made earlier to study the motion of fluid in a curved tube theoretically and experimentally. Most of the work refers to the curved pipe of circular cross section. Srivastava (1980) presented results concerning flow through curved pipe having elliptical cross section and gave a formula in relation to flux ratio. The numerical results presented here could be useful in studying the flow in pulmonary blood vessels, Caro and Saffman (1965) and Systemic Veins Morena etal (1970) as ellipticity has been observed in this region. Caro (1966) carried out experimental work on straight and curved tubes and on a tube containing a short region with an elliptical cross section. The flow regime of interest is based on the parameter of the carotid artery to obtain physically relevant velocity distributions. Bovendeerd etal (1987) indicated that the mean velocity in the common carotid artery is roughly 0.2 m/sec, the radius is about 4 mm and the maximum curvature ratio is about 0.16. It is assumed that blood is a Newtonion fluid with a density of $\rho = 1.132 \times 10^3$ kg m^{-3} at a viscosity of μ=3.56$\times 10^{-3}$ kgm^{-1} s^{-1}. This results in Reynolds number 254.38. This data was endorsed by Verkaik et al (2009).

Dean (1927, 1928) was the first to carry out theoretical analysis concerning flow through curved pipes of circular cross section. He introduced a non dimensional parameter K which associated with his name and called Dean Number.

McConalogue and Srivastava (1968) extended the work of Dean and in their analysis they introduced new dimensional parameter $D = 4R_e\sqrt{\frac{2a}{L}}$, where $R_e$is the Reynolds number, a is the radius of the tube and L is the radius of curvature. The relationship between Dean number K and parameter D introduced by McConalogue and Srivastava is $D = 4\sqrt{K} = 4R_e\sqrt{\frac{2a}{L}}$. The parameter D has it's own importance and has been used extensively by researchers, particularly numerical analysts.

As indicated above for Caroid artery the Reynolds number turns out to be 254.38 and the curvature ratio is 0.16. The parameter $D = 4R_e\sqrt{\frac{a}{L}}$ is then D=576.

II. FORMULATION OF THE PROBLEM

We are considering a curved pipe whose cross section is elliptic. We take A and B as the minor and major axis. Σis the radius of curvature of curved pipe of elliptical cross section.

If Q is the volume flow rate in a curved pipe of elliptic cross section and Q_0 is the volume flow rate in straight pipe of elliptic cross section for the same pressure gradient, then Srivastava (1980) gave the relation

$$\frac{Q}{Q_0} = [1 + \lambda^2 D + \lambda^2 R^2 E + \lambda^2 R_e^4 F] \qquad - (1)$$

where $\lambda = \frac{A}{\Sigma}$, R_e is the Reynolds number, D, E, F are constants and are dependent on the ratio of A/B.

Table-1: The following table gives the values of D, E, F for several values of $C_1 = \frac{A^2}{B^2}$.

Table-1

C_1	D=Coefficient of λ^2	E=Coefficient of $\lambda^2 R_e^2$	E=Coefficient of $\lambda^2 R_e^4$
0.25	–1.7617875	–0.0037665	–0.00001032
0.50	–0.1696069	–0.0019303	–0.00000212
0.75	–0.0052929	–0.0010727	–0.00000038
1	0.0208331	–0.0006334	–0.00000036

We have used equation (1) and Table-1 for computing results which may be useful for medical community. Table-2 gives the values $\frac{Q}{Q_0}$ for different values of D for fixed C_1(1, 0.75, 0.50, 0.25). It may be clarified here that D introduced in the introductory part of the paper is different than what D has been shown in Table-1 and it is this D that is shown in Table-2.

III. RESULTS

Table-2

C_1=1

D	0	50	100	150	200
Q/Q_0	1	0.99424	0.94889	0.78503	0.38173

C_1=0.75

D	0	50	100	170
Q/Q_0	1	0.98167	0.86475	0.17189

C_1=0.50

D	0	50	100	120
Q/Q_0	1	0.95906	0.69216	0.42685

C_1=0.25

D	0	50	65	80
Q/Q_0	1	0.84456	0.7449	0.32969

Table-2 shows that the values of Q/Q_0 decreases for a fixed value of C_1 as the value of D is increased. For each C_1, D has a limit which one gets from the fact that Q/Q_0 should not become negative. May be that for each C_1, one could go little more than what has been shown in the table. We also observe that Q/Q_0 decreases for a fixed value of D as C_1 decreases from C_1=1 to C_1=0.25 i.e. when ellipticity is reduced, there is decrease of flow.

Table-3

λ=0.16, R_e=40

C_1	0.25	0.50	0.75	1
Q/Q_0	0.73307	0.77767	0.90155	0.95100

λ=0.16, R_e=25.5

C_1	0.25	0.50	0.75	1
Q/Q_0	0.78601	0.94062	0.96038	0.98504

λ=0.16, R_e=10

C_1	0.25	0.50	0.75	1
Q/Q_0	0.91886	0.99018	0.99691	0.99799

Table-3 gives the values of Q/Q_0 for the combinations R_e=40, λ=0.16, R_e=25.5, λ=0.16 and R_e=10, λ=0.16 for C_1=0.25, 0.50, 0.75, 1. For each combination Q/Q_0 increases when ellipticity varies from 0.25 to 1.

IV. CONCLUSION

As mentioned in the introduction of the paper that under certain conditions ellipticity develops both in systemic circulation and pulmonary circulation. The knowledge of blood flow in the region where ellipticity develops would be helpful in correctly analyzing the things and to reach a proper conclusion. This paper is concerned with low values of the parameter involved.

McConalogue and Srivastava (1968) numerically solved the problem for higher values of the parameter for flow in curved pipes of circular cross section. This work has application apart from cardiology, in other disciplines as well. This work is being used all over the world for nearly fifty years. Possibly similar solutions need to be obtained for case of flow through curved pipes having elliptical cross section.

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