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OPTIMAL CYCLE TIME DETERMINATION IN INVENTORY SYSTEMS FOR COMPLEMENTARY DETERIORATING PRODUCTS

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ABSTRACT

This research looks at the best way to handle two items that are complimentary and both on the decline in a retail setting. The approach is based on the premise that there is no inventory because all items are sold out as new batches are delivered. Products' pricing, the passage of time, and the influence of cross-demand all factor into exponential functions that control product demand. Using boundary conditions and explicit solutions to differential equations, we determine the product inventory levels. Holding costs, ordering costs, and the effect of buying and selling prices are all part of the cost analysis. Using a linearized version of exponential terms, simplified using Maclaurin's expansion, a profit function is built and the best cycle duration is established. The profit function is checked for concavity using the Hessian matrix to guarantee that it is optimum.

Keywords: *Inventory management, Complementary, Deteriorating, Products, Ordering costs*

I. INTRODUCTION

When it comes to supply chain operations, inventory management is essential since it affects operational efficiency, customer happiness, and company profitability. The restricted shelf life and unavoidable deterioration of goods make inventory management for these items particularly challenging in today's competitive market. Because of the interconnected needs of complementary items, this complexity is magnified. To keep costs down and meet client demands without sacrificing quality, an inventory system for complementing degrading items must take into account demand fluctuations, product price, product deterioration effects, and cost considerations. A slow but steady decline in quality, usefulness, or utility is characteristic of degrading items, which includes foodstuffs, medications, and certain industrial chemicals. Since these goods have a finite shelf life, their fundamental characteristics have a direct effect on inventory levels. Relationships between things provide another level of intricacy when discussing complimentary products. Products that are considered complementary have needs that are interdependent; for example, when sales of one product go up, it usually means that sales of the other go up as well. Printer ink and toner, cellphones and their peripherals, and even bread and butter are all examples of such pairs. Effective inventory strategies for these items can only be designed with a thorough familiarity with their interdependence and degradation dynamics.

A lot of the time, when people think of inventory management, they picture stagnant demand patterns, steady degradation rates, and products with no connections to any other. The mathematical modeling is simplified by these assumptions, but they don't account for the complexities of managing complementary items with unpredictable demand and time-sensitive quality concerns. Advancements in customer-centric operations and cost optimization have prompted academics to create more complicated models that include these factors. These models take into consideration the constraints of product lifecycles and cross-demand impacts to optimize order quantities, cycle durations, pricing strategies, and profit functions. For the purpose of managing supplementary items that are nearing the end of their useful life, zero-inventory solutions have recently come into the spotlight. In order to reduce holding costs and waste, this method makes sure that new inventory arrives only when old inventory is running low. When dealing with expensive or quickly spoiling goods, such solutions are essential to protect profitability. On the other hand, accurate demand forecasting and well-coordinated supply chains are prerequisites for a zero-inventory strategy. The demand for one product might affect the consumption rate and inventory levels of another, further complicating the process. As an example, consider how dependent goods are on each other.

One important tool for dealing with these problems is mathematical modeling. The dynamic behavior of inventory systems may be captured by researchers via the use of models developed using optimization methods and differential equations. For practical insights into inventory management, these models include a number of parameters, including demand elasticity, cross-demand impacts, degradation rates, holding costs, and ordering costs. Case in point: to illustrate the impact of price and time on demand patterns, demand functions for related items often include exponential elements. You can't get inventory level equations, profit functions, or ideal order policies without these demand functions.

It is crucial to have precise demand forecasts while managing complementary items that are degrading. Predicting future demand is essential for several reasons, including setting starting inventory levels, pricing strategies, and order timings. Because of the interaction between the two goods' pricing strategies, demand models must take into consideration the existence of complementary connections. A price cut for one product could increase demand for it and its complementary product at the same time. In contrast, if prices were to rise, fewer people would be willing to buy either product. Maximizing revenue while reducing expenses in inventory systems requires a thorough understanding of these dynamics.

Another important part of managing inventory for items that are very close to expiration is thinking about how much it will cost. Ordering costs comprise procurement and shipping expenditures, whereas holding costs include storage, insurance, and deterioration-related expenditures. The total cost of maintaining a cooperative inventory system may be reduced by economies of scale, which can be achieved through the sharing of ordering expenses. On the other hand, because of the shared expenses, it is essential that the ordering procedures be coordinated so that they match the demand patterns and rates of product degradation. Making ensuring that inventory levels are just right to fulfill client demand without going overboard on prices is what these rules are all about.

An inventory management system's primary goals should be to increase revenue and decrease losses. A mix of price, demand, and cost considerations impact these goals for depreciating supplementary goods. Different pricing tactics may have different impacts on demand, and this is especially true for complementary items, where sales can be greatly affected by cross-demand effects. Optimal pricing choices include striking a balance between making enough money and keeping prices low enough to attract consumers. When it comes to inventory management systems, profit functions provide you the whole picture when it comes to financial performance. They include revenue, holding costs, ordering costs, and buy costs. Those in charge need to think about the consequences of cycle duration to make sure inventory systems for supplementary items that ruin work. Supply and demand, as well as holding and ordering expenses, are affected by the cycle length, which is the duration between subsequent orders. However, holding costs will rise as a result of larger average inventory levels, even if ordering frequency may decrease with longer cycles. In contrast, ordering costs could go up if cycles are too long, while holding costs go down when they are too short. This system's efficiency and profitability are guaranteed by finding the ideal cycle duration, which strikes a balance between these trades-offs.

II. REVIEW OF LITERATURE

Chan, Chi et al., (2017) Finding the optimal production rate to minimize the system's total cost has been under-researched. This study not only offers a way for finding the optimal production rate to use with more traditional models, but it also investigates the effect of production rate on total system cost. The study's findings support a model of an exponentially decaying good with a single vendor and one customer, one that accounts for continuous manufacture and employs production rate as a decision variable. Based on numerical examples, it seems that the proposed model might be a more economical alternative to traditional models that assume a constant product rate. The proposed model accounts for delivery deterioration, which is often overlooked in research on inventory models of deteriorating items. Furthermore, by expanding the proposed model to loosen the assumption of constant cost parameters, it is feasible to optimize the cost for a system whereby a portion of the cost parameters are reliant on the production rate. Even in models where a constant rate of production is not assumed, this assumption remains prevalent.

Pal, Brojeswar et al., (2016) This article discusses production inventory systems for commodities with a near-term expiration date. Production rates in such systems are stochastic within certain bounds, and the unit cost of production is dependent on both the production rate and the size of the manufacturing lot. The approach is based on the premise that products have finite lifespans and will no longer be accessible after that time has elapsed. We are somewhat behind schedule and have permitted shortfalls. The backlog rate is defined by the total amount of time it takes to get a refill. Identifying the optimal production lot size that reduces the inventory system's average anticipated cost per unit time is our top priority. We analytically describe and assess the different cases according to the product lifespan, production runtime, and system cycle length.

Mishra, Vinod Kumar & Singh, Lal. (2011) Disruptions to production are a reality of modern business management. Finding the sweet spot for production time is essential for manufacturers looking to maximize efficiency and profits. In this study, we provide a production inventory model that accounts for time-dependent deterioration items in the presence of production interruption. This model assumes a constant and predictable demand rate and forbids shortages. Additionally, we take into account the reality that the stored item degrades both during regular and interrupted production times. By solving the model analytically, we can get the optimal production time for both normal and disrupted production periods. The solution and application of the model are shown by a numerical example.

Maity, Kanai & Maiti, M. (2009) An inventory management system with degraded multi-items that are either complementary or replacement is defined by a resource constraint. Here, the production function is considered a control variable due to its lack of knowledge. Deterioration rates for items might be continuous or stock dependent. Demand is dictated by stock levels, shortages are undesirable, and used items are reused. For both stable and transient states, we express the total profit as a problem of Pontryagin's Optimal Control. The Generalized Legendre conditions, optimal control theory, and Taylor's theorem are used for its evaluation. Everything from sales to manufacturing costs to inventory holding costs to salvage value goes into this profit.

Maity, Kanai & Maiti, M. (2005) The optimal production strategy for an inventory management system including decomposing multi-items, where items are of complementary or replacement types, is determined by a resource restriction. It is presumed that complimentary items increase demand, whereas alternative products decrease it in a linear fashion. The impact on demand for another item is proportional to the amount of the first item in stock. The goods' rate of degradation is proportional to their availability. Reusing or recycling old items is common practice, inventory levels impact demand, and shortages are never acceptable. Total profit, which includes sales revenues, production costs, inventory holding costs, and salvage value, is determined as a Pontryagin optimal control problem for both steady and transient states, and is assessed using the optimum control theory that fulfills the generalized Legendre condition. For particular numerical data, the model is offered in a generalized form for n-items, and more especially for two items. The ideal results are shown in both tabular and graphical formats.

III. MODEL DEVELOPMENT

This research takes into account a stock-taking strategy in which two goods that are complimentary but are degrading are ordered and sent to the store together, with the order quantity determined by the product's unit cost and the batch ordering cost per order. Because of the system's zero-inventory attribute, the old batch is exhausted just as the new one is about to arrive. As a result, the amounts and begin each cycle with Product 1 and Product 2, respectively. At the conclusion of the cycle, the amount ordered for each product steadily drops to zero owing to degradation and demand. Figure 1 shows the inventory level for the goods throughout the cycle graphically.

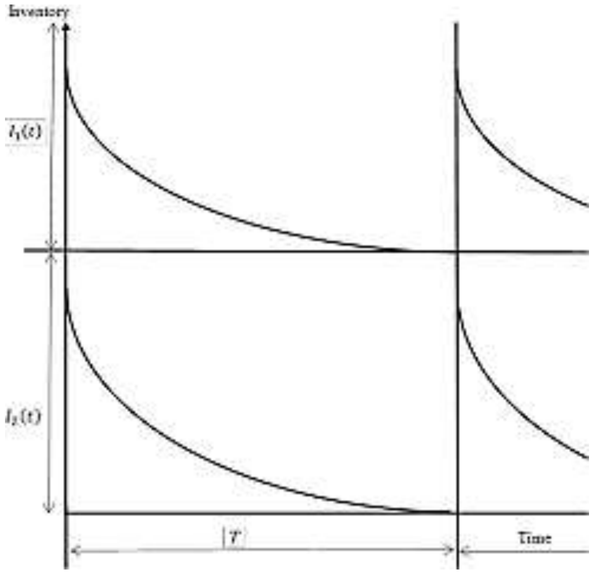

Figure 1: Inventory Dynamics of Two Interdependent Deteriorating Products over Time

The following equations represent the product demand functions:

$D_1 = A_1 e^{-a_1 P_1 - b_1 P_2 - \beta_1 t}$ (1)

And

$D_2 = A_2 e^{-a_2 P_1 - b_2 P_2 - \beta_2 t}$ (2)

At a certain time t and with inventory levels of products 1 and 2, the following differential equations govern the variation in inventory levels.

$\frac{dI_1(t)}{dt} + \theta_1 I_1(t) = -A_1 e^{-a_1 P_1 - b_1 P_2 - \beta_1 t}$ $0 \le t \le T$ (3)

$\frac{dI_2(t)}{dt} + \theta_2 I_2(t) = -A_2 e^{-a_2 P_1 - b_2 P_2 - \beta_2 t}$ $0 \le t \le T$ (4)

The following equations provide the boundary conditions.

$I_1(0) = Q_1$, $I_2(0) = Q_2$ and $I_1(T) = 0$, $I_2(T) = 0$ (5)

Take into account (3) to obtain the following equations for products 1 and 2: $I_1(t)$ and $I_2(t)$ representing their instantaneous inventory levels. The equation for product 1 is as follows.

$\frac{dI_1(t)}{dt} + f(t)y = g(t)$ with integrating factor $I.F = e^{ace10 \int f(t) = e^{\theta_1 t}}$ (6)

the general solution is

$$I_1(t)e^{\theta_1 t} = -A_1 e^{-a_1P_1-b_1P_2} \int e^{t(\theta_1-\beta_1)}\, dt \quad (7)$$

Finding the product 1 inventory level function by integrating Equation (7) yields

$$I_1(t)e^{\theta_1 t} = -\frac{A_1e^{-a_1P_1-b_1P_2}}{\theta_1-\beta_1} e^{t(\theta_1-\beta_1)} + C \quad (8)$$

The inventory level function for product 1 is obtained by solving for C in Equation (8) with the boundary condition $I_1(T) = 0$ and simplifying the equation.

$$I_1(t) = \frac{A_1e^{-a_1P_1-b_1P_2-\beta_1 t}}{\theta_1-\beta_1} e^{(\theta_1-\beta_1)(T-t)} - 1 \quad (9)$$

Similarly, for product 2, we can derive the inventory level function $I_2(t)$ as

$$I_2(t) = \frac{A_2e^{-a_2P_1-b_2P_2-\beta_2 t}}{\theta_2-\beta_2} e^{(\theta_2-\beta_2)(T-t)} - 1 \quad (10)$$

For products 1 and 2, the maximum inventory level functions are obtained by substituting the beginning conditions from Equations (5) into (9) and (10) respectively:

$$I_1(0) = Q_1 \frac{A_1e^{-a_1P_1-b_1P_2}}{\theta_1-\beta_1} e^{(\theta_1-\beta_1)T} - 1 \quad (11)$$

$$I_2(t) = Q_2 \frac{A_2e^{-a_2P_1-b_2P_2}}{\theta_2-\beta_2} e^{(\theta_1-\beta_1)T} - 1 \quad (12)$$

For simplicity let

$$E = A_1 e^{-a_1P_1-b_1P_2} \text{ and } F = A_2 e^{-a_2P_1-b_2P_2} \quad (13)$$

Holding cost

The following formula calculates the overall holding cost for each cycle:

$$\begin{aligned} HC = HC_1 + HC_2 &= h_1 \int_0^T I_1(t)dt + h_2 \int_0^T I_2(t)dt \\ &= h_1 \frac{Ee^{(\theta_1-\beta_1)T)}}{\theta_1-\beta_1}\left[\frac{1-e^{-\theta_1 T}}{\theta_1}\right] - h_1 \frac{E}{\theta_1-\beta_1}\left[\frac{1-e^{-\beta_1 T}}{\beta_1}\right] + \\ & h_2 \frac{Fe^{(\theta_2-\beta_2)T)}}{\theta_2-\beta_2}\left[\frac{1-e^{-\theta_2 T}}{\theta_2}\right] - h_2 \frac{F}{\theta_2-\beta_2}\left[\frac{1-e^{-\beta_2 T}}{\beta_2}\right] \end{aligned} \quad (14)$$

Ordering cost

According to the model, each cycle is assumed to have one order. The following equation represents the shared ordering cost for the two products, which adds up to the total ordering cost:

$$OC = k_0 + \sum_{n=1}^{m} k_n \quad (15)$$

where m = 2 in this case without any loss of generality.

Revenue and Purchase cost of product

The product's revenue is calculated by multiplying the order quantity by the selling price, Q_nP_n, and the purchase cost is calculated in a similar manner, using the unit buy price and the order quantity, $Q_n c_n$.

Profit

The total profit is divided by the cycle length, T, as defined by the following function, to determine profit per unit time:

$$\begin{aligned} TP = \frac{TR - PC - [HC + OC]}{T} = \Bigg[& (P_1 - c_1)\frac{A_1 e^{-a_1P_1 - b_1P_2}}{\theta_1 - \beta_1}\left[e^{(\theta_1-\beta_1)T} - 1\right] + \\ & (P_2 - c_2)\frac{A_2 e^{-a_2P_1 - b_2P_2}}{\theta_2 - \beta_2}\left[e^{(\theta_2-\beta_2)T} - 1\right] \\ & - \Bigg(h_1\frac{Ee^{(\theta_1-\beta_1)T}}{\theta_1 - \beta_1}\left[\frac{1 - e^{-\theta_1 T}}{\theta_1}\right] - h_1\frac{E}{\theta_1 - \beta_1}\left[\frac{1 - e^{-\beta_1 T}}{\beta_1}\right] + \\ & h_2\frac{Fe^{(\theta_2-\beta_2)T)}}{\theta_2 - \beta_2}\left[\frac{1 - e^{-\theta_2 T}}{\theta_2}\right] - h_2\frac{F}{\theta_2 - \beta_2}\left[\frac{1 - e^{-\beta_2 T}}{\beta_2}\right] + k_0 + \sum_{i=1}^{n} k_i)/T \end{aligned} \tag{16}$$

Cycle length

In this part, we will get the ideal cycle time by first linearizing the exponential elements in Equation (16) using Maclaurin's expansion for ex, where x = $(\theta_1 - \beta_1)T$ for simplicity.

$$\begin{aligned} e^{(\theta_1-\beta_1)T} &= \sum_{m=0}^{\infty}\frac{(\theta_1 - \beta_1)^m T^m}{m!} \\ &= 1 + \frac{(\theta_1 - \beta_1)^1 T^1}{1!} + \frac{(\theta_1 - \beta_1)^2 T^2}{2!} + \frac{(\theta_1 - \beta_1)^3 T^3}{3!} \\ &\approx 1 + (\theta_1 - \beta_1)T \end{aligned} \tag{17}$$

We use the approximation from Equation (17) for all the remaining exponential components. What comes out when you plug these estimates into Equation (16) is:

$$\begin{aligned} TP = [& (P_1 - c_1)\frac{E}{\theta_1 - \beta_1}[1 + (\theta_1 - \beta_1)T - 1] + (P_2 - c_2)\frac{F}{\theta_2 - \beta_2}[1 + (\theta_2 - \beta_2)T - 1] - \\ & (h_1\frac{E(1 + (E(1 + (\theta_1 - \beta_1)T)}{\theta_1 - \beta_1}[\frac{1 - (1 - \theta_1 T)}{\theta_1}] - h_1\frac{E}{\theta_1 - \beta_1}[\frac{1 - (1 - \beta_1 T)}{\beta_1}] + \\ & h_2\frac{F(1 + (F(1 + (\theta_2 - \beta_2)T)}{\theta_2 - \beta_2}[\frac{1 - (1 - \theta_2 T)}{\theta_2}] - h_2\frac{F}{\theta_2 - \beta_2}[\frac{1 - (1 - \beta_T)}{\beta_2}] + \\ & k_0 + \sum_{i=1}^{n} k_i)]/T \end{aligned} \tag{18}$$

Simplifying Equation (18) results in:

$$TP = \frac{[(P_1 - c_1)ET + (P_2 - c_2)FT - (h_1ET^2 + h_2FT^2 + k_0 + \sum_{i=1}^{n} k_i)]}{T} \tag{19}$$

By using the law of product differentiation to identify TP′ = (fg)′ = fg′ +f′ gand then equating TP′ = 0 the following is obtained.

$$TP' = (P_1 - c_1)ET^{-1} + (P_2 - c_2)FT^{-1} - (2h_1ET + 2h_2FT)T^{-1} - (P_1 - c_1)ET^{-1} - (P_2 - c_2)FT^{-1} - \left(-h_1ET^0 - h_2FT^0 - k_0T^{-2} - \sum_{i=1}^{n} k_iT^{-2}\right) = 0 \tag{20}$$

Solving Equation (20) for T to find the optimum T yields:

$$T = \sqrt{\frac{k_0 + \sum_{i=1}^{n} k_i}{h_1E + h_2F}} \tag{21}$$

where E and F have been defined in Equation (11) for simplicity.

Proof of optimality

Evidence that the profit function's Hessian matrix is negative (semi)definite. The unit profit function TP is shown to be concave using Equation (19).

The Hessian matrix for TP is given by H (P_1, P_2, T) =

$$\begin{pmatrix} \frac{d^2TP}{dP_1{}^2} & \frac{d^2TP}{dP_1dP_2} & \frac{d^2TP}{dP_1dT} \\ \frac{d^2TP}{dP_2dP_1} & \frac{d^2TP}{dP_2{}^2} & \frac{d^2TP}{dP_2dT} \\ \frac{d^2TP}{dTdP_1} & \frac{d^2TP}{dTdP_2} & \frac{d^2TP}{dT^2} \end{pmatrix} \tag{22}$$

IV. NUMERICAL RESULT

Take into account a store setting for two consumables that are complementary to one another and whose parameter values are listed in Table 1. This problem's solution is given.

Table 1: Parameter Values

Parameters	Product 1	Product 2
a_n	0.06	0.02
b_n	0.04	0.08
n	0.40	0.50
h_n	6.00	5.05
c_n	24.00	20.02
A_n	300	450
n	0.28	0.35
e	2.718	-
k_0	500	-
K_n	150	300

The profit function is negative (semi)definite since the following requirements are met, as evidenced by the first test of the optimality condition: $\varepsilon = (P_2 - c_2) - Th_2 = (33.92 -$

20.02)−3.06×5.05 = −1.14 < 0 and $\omega = (P_1 - c_1) - Th_1 = (40.09 - 24) - 3.06 \times 6.00 = -2.21 < 0$. The next step in fixing the issue was to use the values from Table 1 to replace the parameters in the model's relevant equations. The outcomes shown in Table 2 are the consequence of applying the appropriate model equations in Excel.

Table 2: Model results

P1	**P2**	**T**	**T P**	**Q1**	**Q2**
40.09	33.92	3.06	1234.95	122.60	223.39

V. CONCLUSION

Optimal cost management, prompt replenishment, and accurate demand forecasting are the three pillars upon which the inventory management of two degrading items that work in tandem rests. To overcome these obstacles, the suggested model incorporates degradation rates, demand dependency, and zero-inventory principles to reduce holding costs and waste. The paper shows how strategic inventory strategies may improve operational efficiency and profitability through proven numerical analysis and exact mathematical formulations. This method is in line with current company goals of minimizing resource wastage and increasing value, and it also guarantees smooth operations across the supply chain. Adopting such streamlined inventory strategies will be critical for firms to preserve competitiveness and drive long-term growth as they traverse shifting market conditions.

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