

EXPERIMENTAL INVESTIGATION USING VIBROMETER AND MODAL ANALYSIS OF CANTILEVER BEAM

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ABSTRACT

Modal analysis has become a major technique to determine dynamic characteristics of engineering structures and its components. It is a process by which the natural frequencies, mode shapes and damping factor of structures can be determined with a relative ease. In the present work for a cantilever beam the natural frequencies and mode shapes are obtained through experimental, analytical and numerical method. The natural frequencies through experiments are determined using a laser vibrometer, for which the beam is fixed at one end and the free vibrations of the beam are excited using impact hammer. Thus obtained natural frequencies and mode shapes are compared with the results obtained through finite element modeling using ANSYS Workbench 14.5 and also with the analytical results. A good correlation between the all three i.e. numerical results, experimental results and theoretical results are observed.

Keyword: Cantilever beam, finite element modeling, impact hammer, laser vibrometer, modal analysis

I. INTRODUCTION

The experimental study for engineering component always provided a major contribution to our efforts to control and understand the many vibration phenomena's encountered in practice [1]. The modal analysis process has two types of methods to analyse the structures. First is theoretical modal analysis and second is experimental modal analysis. In the theoretical modal analysis method (fig.1) the spatial properties of the model (mass, stiffness and damping) are given and using them modal model and response model are obtained.

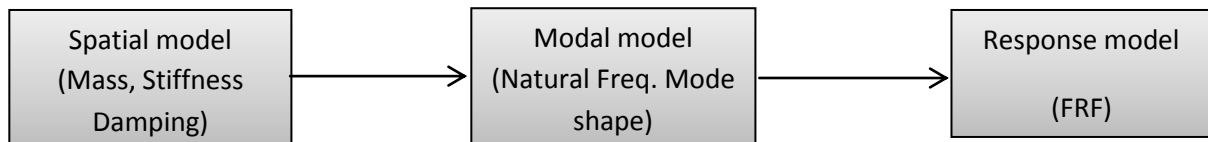


Fig: 1 Theoretical route to vibration analysis

In theoretical modal analysis one cannot forecast accurate boundary conditions, actual rigidity and damping for complex engineering structures and components. So the calculated results often have certain error with actual results [1, 2]. In the 1940s, the experimental modal analysis was first developed to investigate the dynamic behaviours of structures using the simple sine dwell method. In the past two decades, a method for modal analysis based on Fast Fourier Transformation (FFT) approach has received the widest acceptance. The

experimental modal analysis method enables engineers to get a better understanding of dynamic properties of structures [3]. It is a path from response model to modal model (fig 2) and not so often ending with spatial model. Experimental modal analysis used to derive the modal model of alinear time-invariant vibratory system.

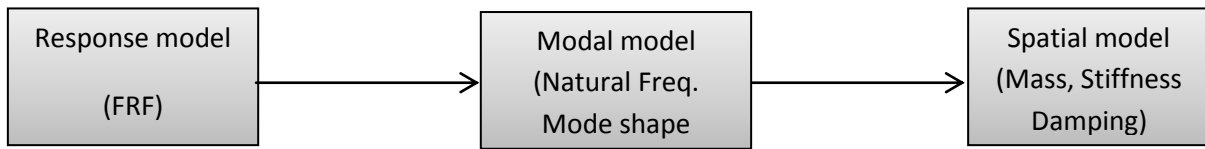


Fig. 2 Experimental route to vibration analysis

Experimental modal analysis using vibrometer is Non- destructive testing, based on vibration response of the structures. For excitation of structure impact hammer is widely used in modal analysis [4]. It is well known thatfor (mechanical) structures falling under resonant conditionssmall force can result in large deformation, and possibly, damage can be induced in the structure. The interaction between the inertial and elastic properties of the materialscauses resonant vibration in the structure [5]. Modal analysis is frequently used to find mode of vibrations of machine component and structure.

In this paper, the modal parameters i.e. natural frequencies, and mode shapes for the cantilever beam are determined experimentally using laser vibrometer. The results of thus obtained natural frequencies are then compared with theoretically and numerically calculated values.

II. THEORETICAL MODAL ANALYSIS OF BEAM

Consider an Euler-Bernoulli uniform cantilever beamundergoing transverse vibrationcondition as shown in Fig.3.

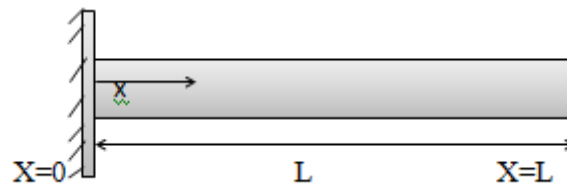


Fig.3Cantilever beam

For free vibrations the equation of motion of beam can be given as [6]

$$EI \frac{d^4w}{\partial x^4} + \rho A \frac{\partial^2 w}{\partial t^2} = 0 \quad (1)$$

$$c^2 \frac{d^4w}{\partial x^4} + \frac{\partial^2 w}{\partial t^2} = 0 \quad (2)$$

$$\text{where } c = \sqrt{\frac{EI}{\rho A}} \quad (3)$$

Free vibration solution can be found using the method of separation of variable. Assuming solution as $w(x, t) = w(x)T(t)$ (4)

From equation (2) and (4)

$$\frac{c^2}{w} \frac{d^4 w}{dx^4} T = -\frac{1}{T} \frac{d^2 T}{dt^2} = \alpha \quad (5)$$

Where $\alpha = \omega^2$ can be shown as a constant. The equation (5) can be written as two equations

$$\frac{d^4 w(x)}{dx^4} - \beta^4 w(x) = 0 \quad (6)$$

Where

$$\beta^4 = \frac{\rho A \omega^2}{EI} \quad (7)$$

$$\frac{d^2 T(t)}{dt^2} + \omega^2 T(t) = 0 \quad (8)$$

From the equation (7) the natural frequency of beam ω can be written as

$$\omega = (\beta L)^2 \sqrt{\frac{EI}{\rho A L^2}} \quad (9)$$

The solution of equation (8) is

$$T(t) = A \cos \omega t + B \sin \omega t \quad (10)$$

Where A and B are constant that can be determined from the initial boundary conditions.

Assuming the solution of equation (6) as

$$W(x) = C e^{\beta x} \quad (11)$$

Using (6) and (11) one can obtain the general solution

$$W(x) = C_1 (\cos \beta x + \cosh \beta x) + C_2 (\cos \beta x - \cosh \beta x) \\ + C_3 (\sin \beta x + \sinh \beta x) + C_4 (\sin \beta x - \sinh \beta x) \quad (12)$$

Where, the constants C_1, C_2, C_3 and C_4 can be determined from the boundary conditions. For a cantilever beam the transverse deflection and its slope must be zero at the fixed end and at free end the bending moment and shear force must be zero.

Thus the boundary conditions become

$$W(0) = 0 \quad (13)$$

$$\frac{dW}{dx}(0) = 0 \quad (14)$$

$$\frac{d^2 W}{dx^2}(L) = 0 \quad (15)$$

$$\frac{d^3 W}{dx^3}(L) = 0 \quad (16)$$

Substituting the equation from (13) to (16) in equation (12) to obtain

$$\cos\beta L + \cosh\beta L + 1 = 0 \quad (17)$$

Equation (17) is the frequency equation. This transcendental equation can be solved to obtain the value of $(\beta L)^2$ and for the cantilever beam the values are given in Table 1[5].

Table: 2 The value of $(\beta L)^2$

Mode	$(\beta L)^2$
Mode 1	1.875104
Mode 2	4.694091
Mode 3	7.854757

The equation (9) for circular natural frequency of cantilever beam in rad/sec this value of frequency is converted to Hz by dividing ω with 2π . The material and geometric parameter used for theoretical and numerical analysis of beam are tabulated in Table 2.

Table: 2 Geometric and Material Parameter of Beam

Material Parameter	Geometric Parameter
$E = 2.05 \times 10^{11} N/m^2$	$L = 0.7m$
$\rho = 7830 kg/m^3$	$B = 0.025m$
$\vartheta = 0.3$	$t = 0.004m$

III. EXPERIMENTAL MODAL ANALYSIS OF BEAM

To verify the theoretical and numerical results experimental setup is prepared. The experimental setup used is shown in fig. 4. The cantilever beam is manufactured in workshop and the material used for beam is mild steel. The cantilever beam is excited with the help of impact hammer. The vibration of beam is measured with the help of laser vibrometer (Polytech laser vibrometer NLV-2500).



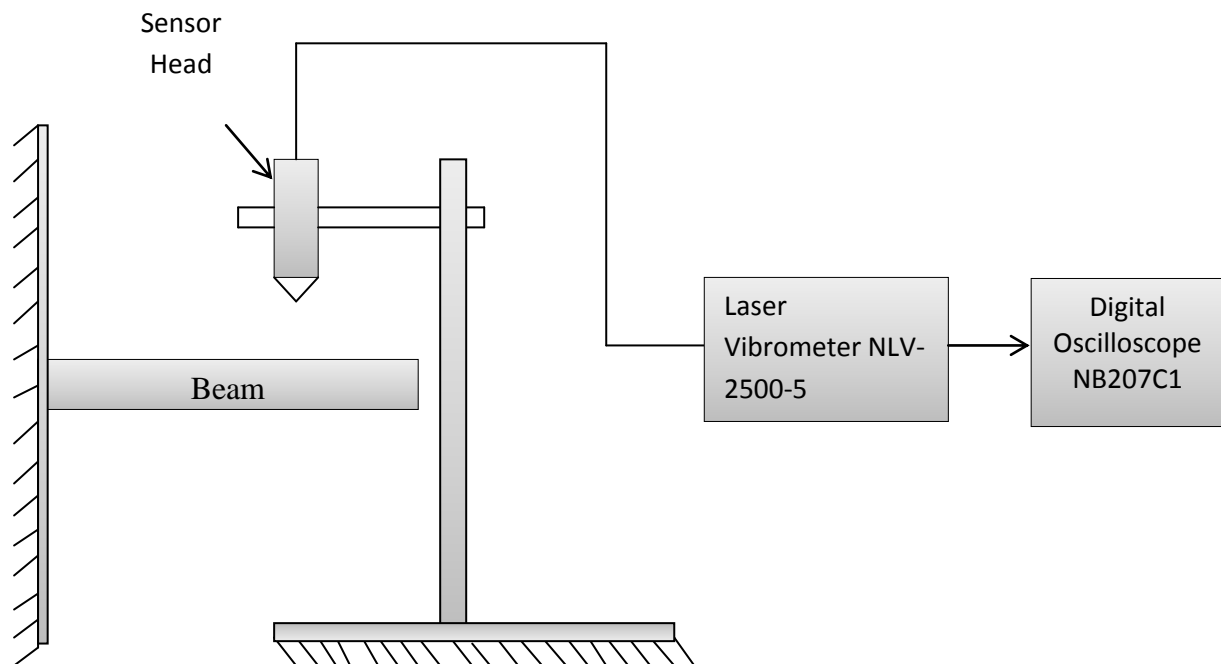


Fig: 4 Experimental setup

The laser vibrometer has two unit-one is electronic unit which process the signal and supplies laser signal the other is laser sensor head having CCD camera for better visualization. Finally the excitation signal from the laser vibrometer are fed to a digital oscilloscope (NLB207C1) in which the data is processed. The signal in time domain is saved in USB drive from the digital scope. The time domain signal is converted in to frequency domain using Fast Fourier Transformation with the help of MATLAB program.

IV. NUMERICALLY MODAL ANALYSIS OF BEAM

The three-dimensional finite element model of cantilever beam is constructed in ANSYS Workbench 14.5 and then computational modal analysis is performed to generate natural frequencies and mode shapes. The geometric and material parameter is taken from the Table 2 [8].Solid 185 element are adopted for cantilever beam analysis. Relevant boundary conditions are applied at the fixed end of cantilever beam [9]. The numerically results are compared to theoretical and experimental results.

V. RESULTS AND DISCUSSION

The results of the theoretical natural frequencies and numerically obtained natural frequencies of mild steel, cantilever beam are calculated using the material properties and dimensions of the beam given in Table 2. The theoretical natural frequencies are calculated using the equation (9) and the finite element natural frequencies are determined using ANSYS Workbench 14.5. Table 3 depicts the theoretical, numerical and experimental natural frequencies of the beam. To validate the theoretical and the numerical model extensive tests were carried out at two natural frequencies as shown in Table 3.

Table: 3 theoretically, numerically and experimentally natural frequency of beam of beam

Mode	Theoretically Natural frequency (Hz)	Numerically Natural frequency (Hz)	Experimentally Natural frequency (Hz)
Mode 1	6.73	6.76	6.71
Mode 2	42.22	42.38	39.06
Mode 3	118.22	118.67	---

The experimental obtained time domain signal and frequency domain signals are shown in fig. 5.

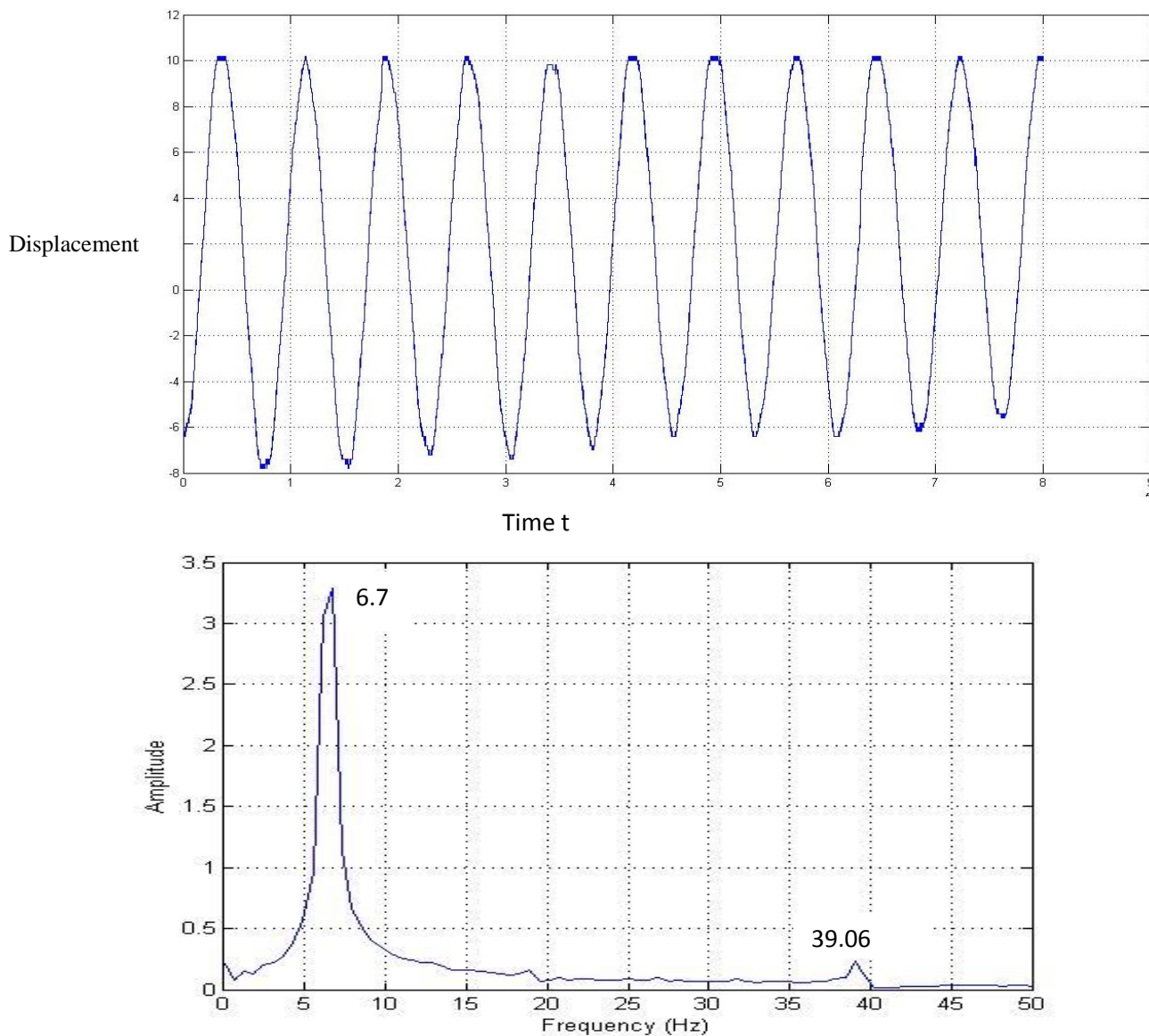


Fig. 5 Time domain and frequency domain signal

We can see in frequency domain graph the first peak point occurs at the first natural frequency (6.71 Hz) and the second peak is at the second natural frequency (39.06 Hz). The time domain signal is converted in frequency

domain signal using Fast Fourier Transformation algorithm with the help of MATLAB program. The numerical mode shape and corresponding natural frequency obtained using ANSYS are shown in fig.6.

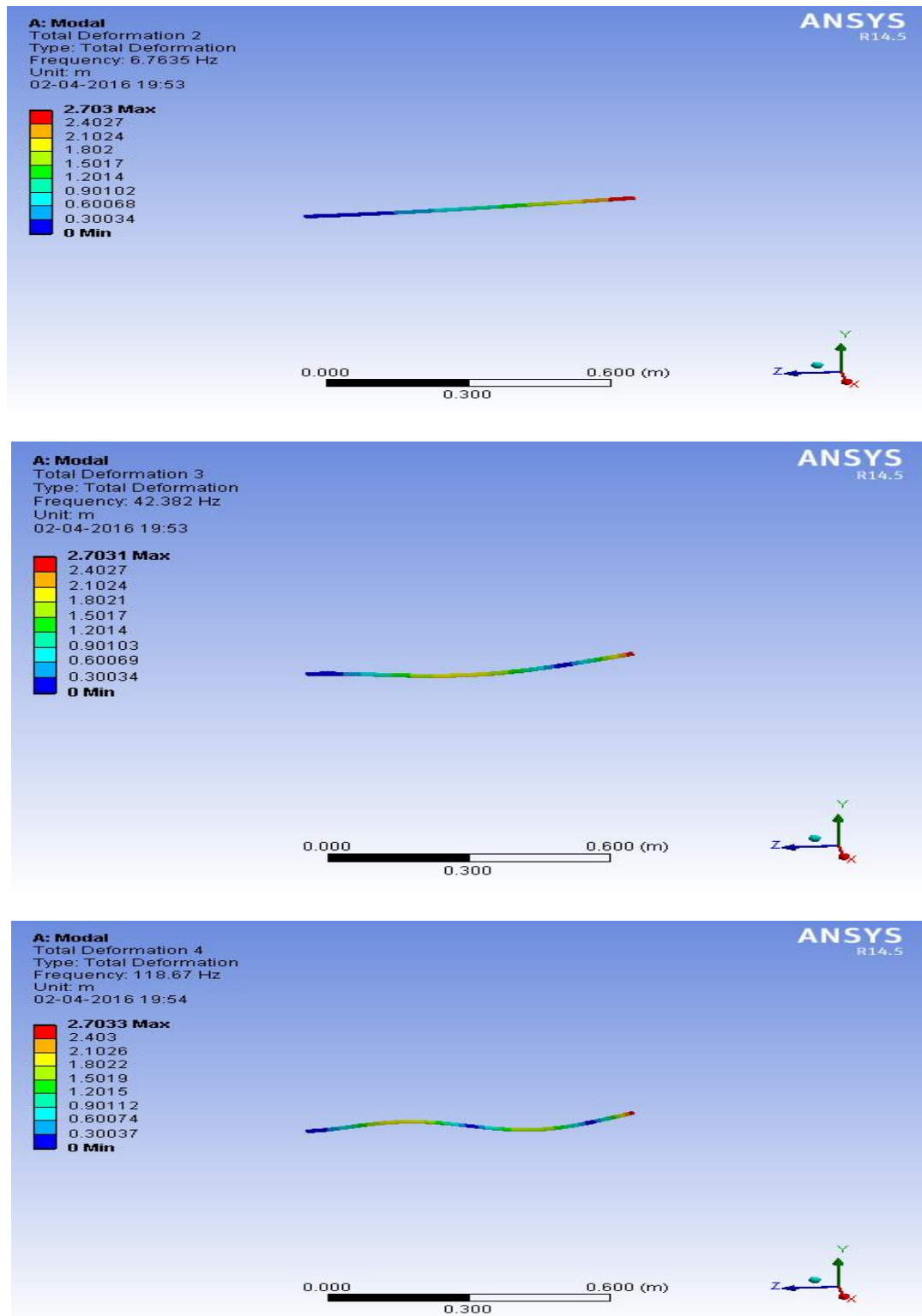


Fig. 6 Mode shape and corresponding natural frequency of cantilever beam

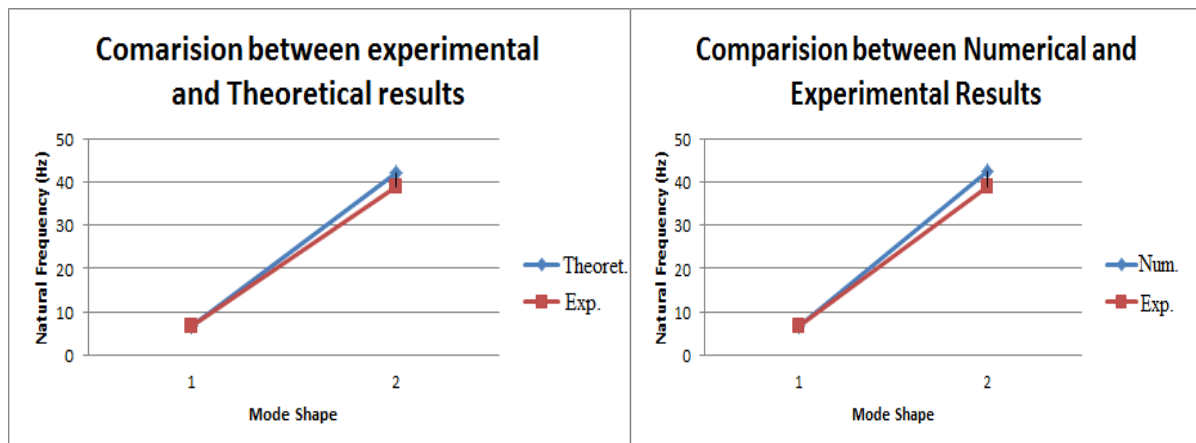


Fig. 7 Comparison Graph between numerical, theoretical and experimental

Fig. 7 displays the comparison of the first two natural frequencies obtained using various methods. It can be observed that in the experiment it was not possible to excite the third mode using impact hammer. For determination of higher natural frequencies the sine sweep test can be more effective.

VI. CONCLUSION

In this paper the theoretical, experimental and numerical modal analysis of cantilever beam are performed. Experimental modal analysis of the beam was carried out using laser vibrometer and impact hammer excitation. The numerical results are obtained using ANSYS Workbench 14.5. The experimental, numerical and theoretical results are found to have extremely good correlation. Some error between the results is also introduced by physical dimensions of cantilever beam and variation of young's modulus. The overall analysis show that the results are all well within reasonable error.

The experimental modal analysis using laser vibrometer is found to be an effective method to determine the dynamics properties of beam. However, the higher modes of the beam can't be excited using impact hammer.

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