

FINITE ELEMENT ANALYSIS OF PIEZOELECTRIC TRANSDUCERS

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ABSTRACT

In the project work the main emphasis is to be given to the analysis of a transducer which can be used as a sensor to calculate the dynamic loading i.e. to calculate the frequency and load in dynamic loading. Besides that, the work has been divided in different sections which include the modeling of various types of transducers that can be used for dynamic loading. For the project work, only a particular piezoelectric material, PZT-4, is to be considered, which is most widely used in sensors and actuators.

Index Terms: - Piezoelectricity, Pzt-4, Piezoelectric Transducer

I. INTRODUCTION

The word *piezoelectricity* literally means ‘pressure electricity’; the prefix *piezo* is derived from the Greek word *piezein* which means “to press”. Piezoelectricity is the property of a crystal by which electric polarization is produced by mechanical strain in crystal and conversely production of strain on application of voltage. The electric polarization produced is proportional to the applied strain and changing direction with it. Piezoelectricity is different from electrostriction, another effect which causes a solid dielectric to change shape on application of a voltage. In piezoelectricity, a reversal of voltage reverses the sign of the resulting strain whereas for electrostriction the strain is an even function of the applied voltage.

Piezoelectric transducers have been extensively applied over the last decade to diverse area such as band-pass filters and high-energy ultrasonic devices. They are typically utilized for generating an acoustic pulse proportional to the input external loading and detecting the same and converting it in to electric signal. Different other application of piezoelectric are in loudspeaker, microphone, phonograph, wrist watches, oscillator, producing very selective filter circuit, resonator, transducer etc. Piezoelectric crystals are widely used in aerospace industry, automobile industry and geological instruments as sensors and actuators.

II. OBJECTIVES

In the project work the main emphasis is to be given to the analysis of a transducer which can be used as a sensor to calculate the dynamic loading i.e. to calculate the frequency and load in dynamic loading. Besides that, the work has been divided in different sections which include the modeling of various types of transducers that can be used for dynamic loading. For the project work, only a particular piezoelectric material, PZT-4, is to be considered, which is most widely used in sensors and actuators. Following are to be the main objectives of the study –

Finite element modeling of cantilever beam made of piezoelectric material PZT- 4, determination of Eigen values and corresponding mode shapes including the analysis of electrical output at various frequencies.

1. Model of simple beam made of piezoelectric (PZT-4)
2. Model of piezoelectric shell with Brass end cap

3. Model of beam with piezoelectric element.
4. Model of Proving Ring with Piezoelectric Strip
5. Piezoelectric Strip.

III. MODELING AND ANALYSIS

In the modeling and analysis part of this project, the work has been performed in different phases which include the learning of the ABAQUS i.e. how to use ABAQUS for the analysis of piezoelectric material. Earlier version of ABAQUS were not having the facility to deal with piezoelectric properties but the version 6.5-1 has the facility to analyze the structures which are having the piezoelectric parts, but with some limitations. Before performing the following studies, I have also gone through the examples available in ABAQUS documentation but there are some limitations in this version which was found while solving the problems related to piezoelectric. For example its explicit analysis does not have the piezoelectric elements. Even though I performed the following studies in best possible way with the facilities available in ABAQUS 6.5-1 . The work has been arranged as described in chapter3, and the analysis has been performed for dynamic loading. A separate analysis has been done to understand the behaviour of piezoelectric response with the change in temperature.

There is no separate heading for the analysis and result, all the results are shown with the analysis and discussed also. After each study , the importance is also described.

3.1 Finite element modeling of cantilever beam made of piezoelectric material pzt- 4 and determination of eigen values and corresponding mode shapes.

Modeling

To start with the piezoelectric transducer, first take the simplest case i.e. a cantilever beam having piezoelectric material. To understand the nature of piezoelectric as a single material this condition has been taken. To start the analysis we divide the real structures in three parts , beam, plate and shell. Model of the first analysis has been prepared as shown in Fig 4.1.

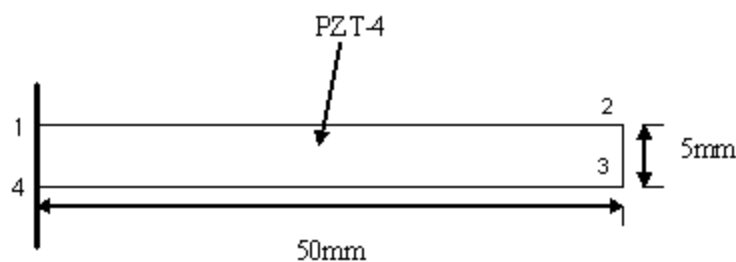


Fig 4.1. Model of simple beam made of piezoelectric (PZT-4)

In the above problem the material used is PZT-4. The main purpose of the analysis is to learn about the proper modeling of the piezoelectric material structures. Generally for a simple material we are having only 6 degree of freedoms but for the material like piezoelectric we have one more degree of freedom which is directly related to the electrical properties of the piezoelectric material. In stead of the eigen values and corresponding mode shapes , analysis of electric potential at points 1,2,3,4(fig4.1.1) is also done.

Element Type

CPE4E – A 4 node bilinear plain strain piezoelectric quadrilateral.

Material

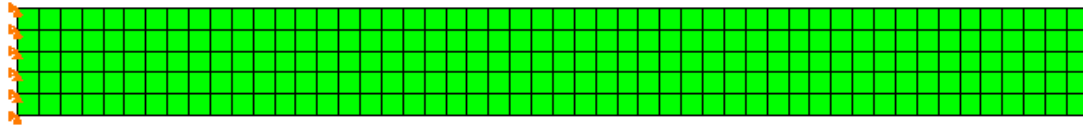


Fig 4.2 Meshed Cantilever made of Piezoelectric PZT-4

3.1.1 Results and discussion

Other than the eigen values and the mode shapes, the voltage output of the beam is also calculated at different points.

This output is shown in table 4.1

Electrical potential output

Frequency(cycle/time)	EPOT-1	EPOT-2	EPOT-3	EPOT-4
1031.9	-8.838	-8.846	-8.846	-8.854
6259	5.056	5.003	5.003	4.949
16515	-0.4056	-0.5612	-0.5612	-0.7168
20726	35.7	38.68	38.68	35.7
29990	-4.829	-4.522	-4.522	-4.215
45594	-22.48	-22.98	-22.98	-23.48
62059	33.65	36.59	36.59	3.65

Table 4.1 Electrical potential at point 1,2,3,4 with frequencies.

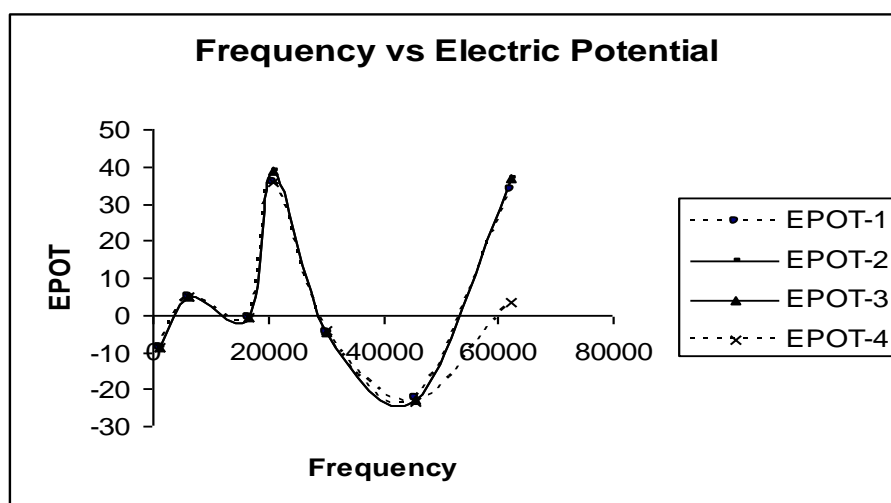


Fig 4.3 Graphical representation of frequency vs. electric potential

On the basis of above results we find that the mode shapes are totally dependent upon the natural frequency. Being the continuous beam, it will have infinite no of eigen values and corresponding mode shapes. If we observe carefully from fig 4.4 then there is a sequential shape change in the beam mode shapes, continuously the no of maxima & minima is increasing by one in each eigen values. This shows that as frequency increases, there is correspondingly more irregu-

lar mode shape between two points. Also if we see the output voltage then we see that for a particular frequency, the potential developed in all the corners is same which indirectly shows that on changing frequency the corresponding electric potential developed at each point in the piezoelectric transducer will be almost same.

Thus by the above methodology it is possible to make a beam of pure piezoelectric material which can be used as a transducer. Such a beam can be similarly used as the strain gauges to measure the strain. The difference is that we will get the electrical response in stead of mechanical response. For this a proper calibration is needed.

3.2 Finite element modeling of cylindrical shell made of PZT-4 and having end cover of Brass and determination of eigenvalues and mode shapes

Modeling

The idea behind the solving such situation was to have a good understanding about the assembly of piezoelectric with the other materials. Also to understand the effect of the piezoelectric material while it is in the combination of other material.

The modeling of this type of shell is done by taking 2-D asymmetric element. The upper half of the diagram as shown in following figure is constructed. And then analysis is done

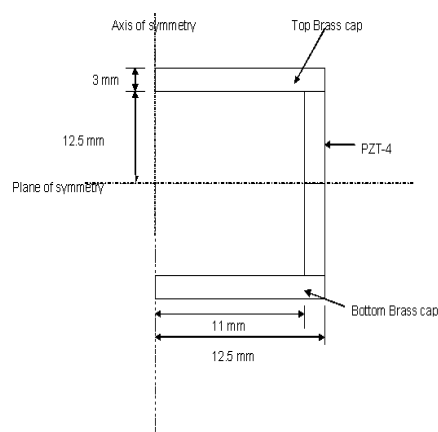


Fig- 4.5 Model of piezoelectric shell with Brass end cap

Considering the following:-

Element type

Brass

CAX4R: A 4 node bilinear axisymmetric quadrilateral

Total no of elements- 600

Total no of nodes- 656

PZT-4

CAX8RE: An 8 node biquadratic axisymmetric piezoelectric quadrilateral

Total no of elements- 600

Total no of nodes- 1941

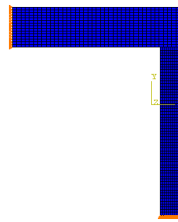


Fig: 4.6 The mesh representing the shell element in axisymmetric Form

Boundary Condition

At axis of symmetry no displacement in radial direction

At plane of symmetry no displacement in vertical direction

3.2.1 Result and discussion

Mode shapes

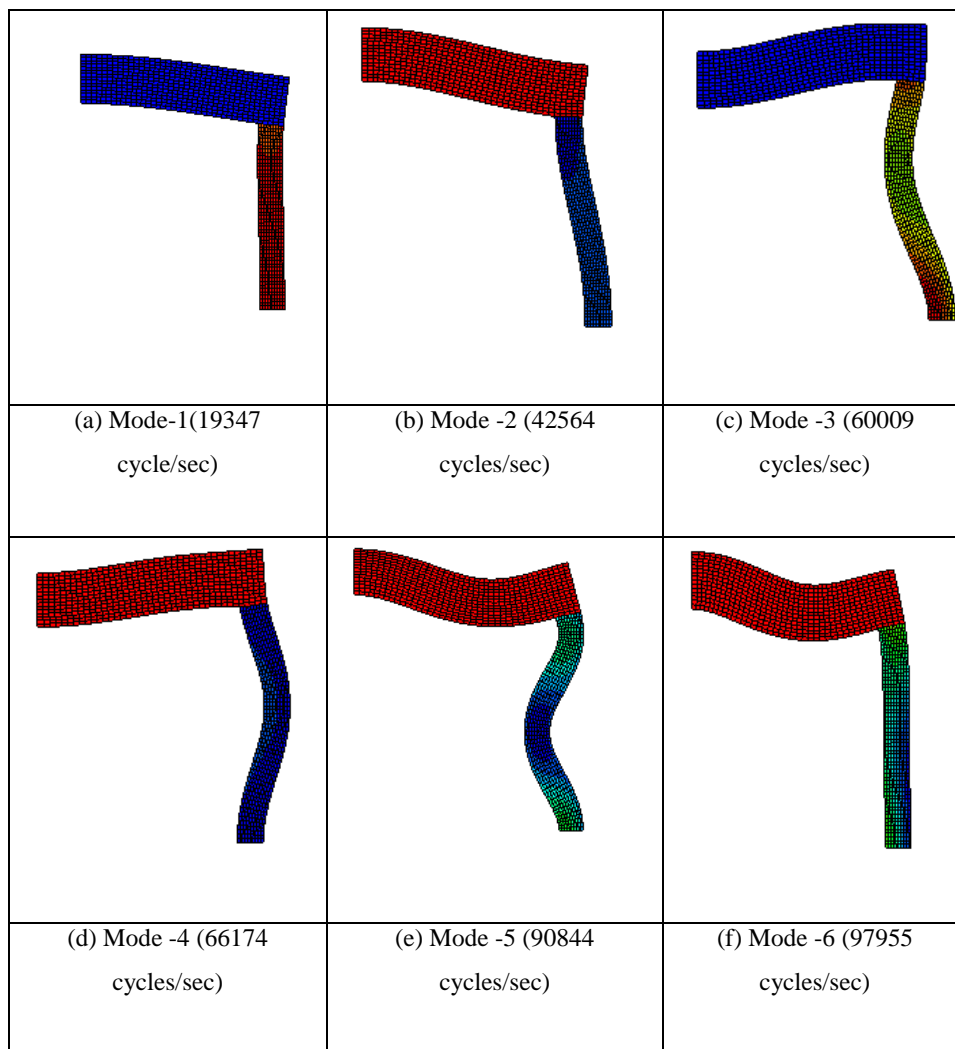


Fig 4.7 Mode shapes of the model for first 6 Eigen values

Electric potential output

In this problem the electrical potential is calculated at the plane of symmetry of the shell.

Sr. NO.	Frequency(cycles/sec)	Electric potential
1	19347	2.958
2	45564	-7.174
3	60009	4.531
4	66174	-2.606
5	90844	-4.034
6	97955	-9.35
7	118651	-6.565

Table 4.2 Electrical potential at plane of symmetry with frequencies.

The mode shapes shown above just represent the mode shapes while no loading is there. If we will apply the boundary conditions in terms of potential difference or electric charge then a different mode shape will be there. The electrical output will also change. That effect we will consider in forthcoming problem in the same work.

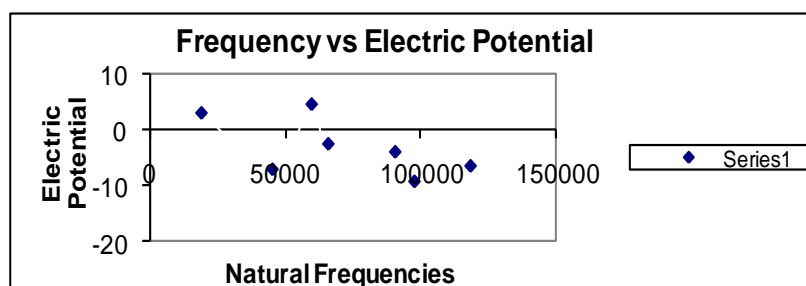


Fig 4.8 Graphical representation of frequency vs. electric potential

After going through the result with respect to the varying frequency, we noticed that in 1st, 2nd and 4th mode the electric potential developed is same throughout the piezoelectric element, but in other three cases we see that the electric potential is different for the different nodes. Thus while analyzing the piezoelectric transducer we have to take care about frequency also.

IV. RESPONSE UNDER DYNAMIC LOADING

The same model has been analyzed under the dynamic loading with the sinusoidal loading of frequency 250 cycles/second and load varying from 20N to 200 N.

After going through the result, find that the output flux is increasing with the increase in the amplitude of load but after some time the increase in load is not with the same rate as in the beginning. Thus we can say that such a transducer possible to be used for the determination of amplitude of dynamic loading, if frequency is already known, the only thing required is proper calibration. All this give us an idea to make such transducer to be used for dynamic loading and these can be installed anywhere in a big structure for the same purpose.

4.1 Finite element modeling of cantilever beam with piezoelectric transducer pzt-4 and determination of its response under sinusoidal loading

Modeling

To solve this problem we consider a 2-D planer cantilever beam fixed at one end. In this cantilever beam we attach a piezoelectric element (PZT-4) as shown in figure with the TIE option The cantilever beam is kept at zero potential difference. A periodic load is applied on the right most corner with different frequencies.

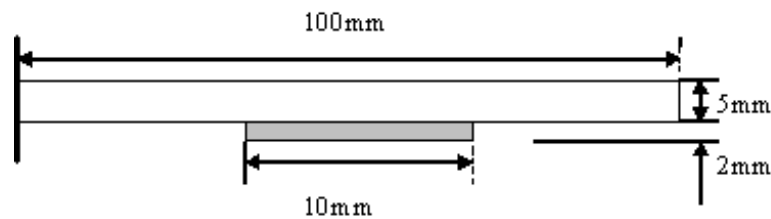


Fig 4.19 Model of beam with piezoelectric element.

Material

Piezoelectric Element - PZT-4

Element

Beam

CPS4R – A 4 node bilinear plane stress quadrilateral.

Total no of elements- 322

Total no of nodes – 504

Transducer

CPS4E - A 4 node bilinear plane stress piezoelectric quadrilateral.

Total no of elements- 80

Total no of nodes-105

Boundary condition

Rigidly fixed i.e. $U_1 = U_2 = U_3 = 0$

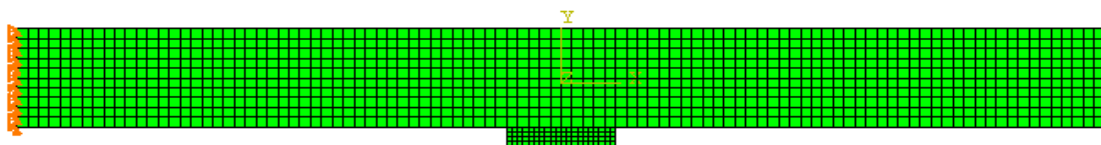


Fig: 4.20 Meshed structures of Beam and Transducer

V. RESULT AND DISCUSSION

Following is the table of the electrical output with respect to the natural frequency of the system.

Frequency	EPOT	Frequency	EPOT
671.1	19.01	6040	22.36
1342	69.18	6711	7.918
2013	17.22	7383	9.147
2685	18.74	8054	1.076
3356	20.73	8725	0.432
4698	76.94	9396	10.22
5369	3.262		

Table 4.3. Frequency Vs Electric Potential in Piezoelectric Element

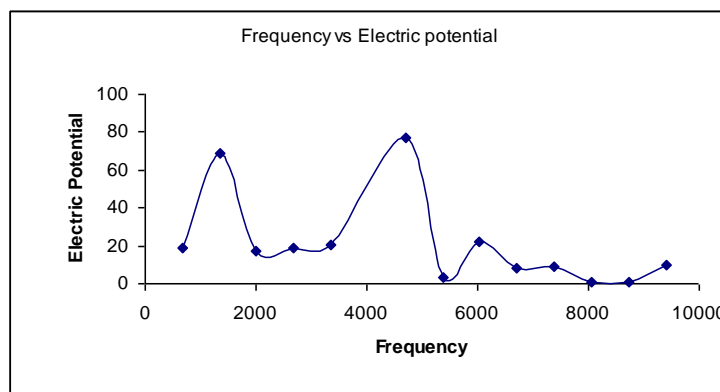


Fig: 4.21 Frequency Vs output voltage

From the graph above shown, we can see that there is almost an uniform output. But suddenly at a point the output voltage is very-very large then the others this is because the resonance frequency may occur near this point. Otherwise there is not much difference in the output with respect to the frequency under periodic loading.

Table 4.4 Load Vs Electrical flux output while frequency is constant

S.No.	Frquency (Cycles/sec)	Load (N)	Maximum Electric Flux
1	250	10	6.411×10^{-6}
2	250	20	1.119×10^{-5}
3	250	30	1.678×10^{-5}
4	250	40	2.237×10^{-5}
5	250	50	2.796×10^{-5}
6	250	60	3.356×10^{-5}
7	250	70	3.915×10^{-5}
8	250	80	4.474×10^{-5}
9	250	90	5.034×10^{-5}
10	250	100	5.593×10^{-5}
11	250	110	6.152×10^{-5}
12	250	120	6.711×10^{-5}

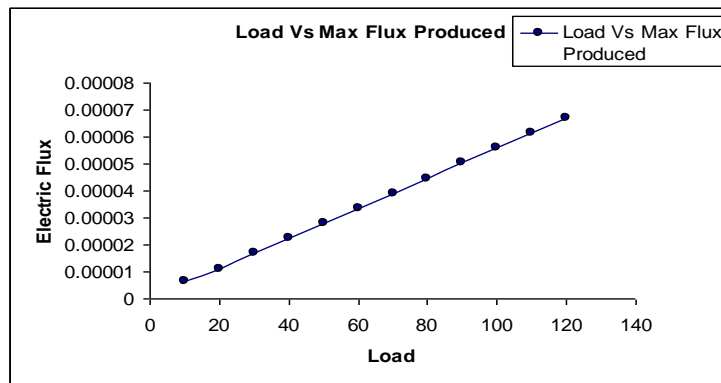


Fig: 4.22 Load Vs Max Flux Produced

5.1 Finite element modeling of proving ring with piezoelectric layers as a sensor and dynamic analysis under sinusoidal loading with the variation in frequency with constant load and with the variation of load under constant frequency.

Proving ring is a device which is used to calibrate the instruments which apply the static loading eg. Universal Testing Machine etc. The following model is presented with an expectation that the same ring can be used to calibrate the instruments involving the dynamic loading if we use the piezoelectric strips as a sensor.

Modeling

The modeling has been done as follows. It contains five parts, one is the ring made of stainless steel and rest of the four parts is made of piezoelectric material PZT-4. The steel ring is the main body and the rest of the four parts are made as integral part of the main ring with TIE option. The construction is as per the following diagram.

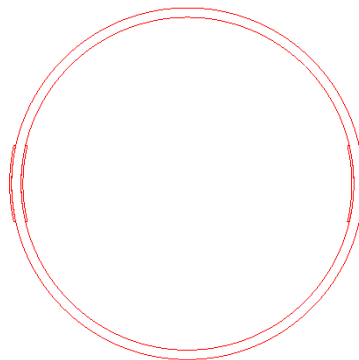


Fig 4.23 Model of Proving Ring with Piezoelectric Strip

Now each part will be described individually. The first part is the ring made of stainless steel as shown in the following figure. The steel ring is modeled as followed-

Inner Diameter – 175mm

Outer Diameter – 185mm

No. of elements – 70

No. of nodes - 350

Element type – CPS8R

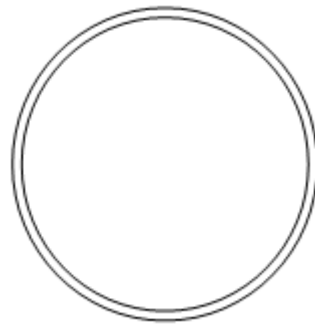


Fig 4.24 Ring made of Stainless steel

Other four part are made of piezoelectric material PZT-4. All the four strips are having thickness of 1mm. All the strips are modeled as follows

No. of elements – 40

No. of nodes - 203

Element type – CPS8RE

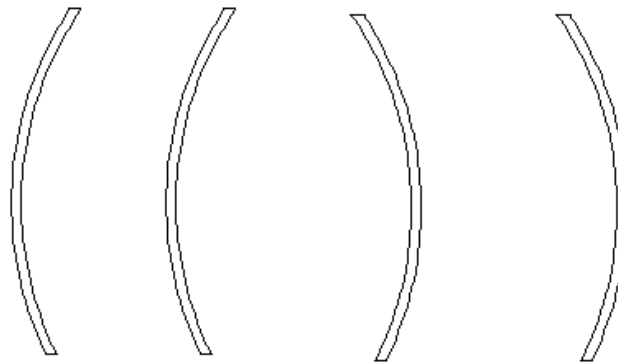


Fig 4.25 Piezoelectric Strips

5.1.1 Static Analysis

After the modeling of the transducer, the first part of the analysis is static analysis. In the part of static analysis, the analysis is done by applying the static load on the top point in the downward direction. The range of load applied is 20N to 500 N. The result of the static analysis is shown in the Table 4.5

S. No.	Load	Max. Strain	Plastic Strain
1	20	6.937×10^{-12}	0
2	40	1.137×10^{-11}	0
3	60	2.081×10^{-11}	0
4	80	2.775×10^{-11}	0
5	100	3.469×10^{-11}	0
6	150	5.203×10^{-11}	0
7	200	6.937×10^{-11}	0
8	250	8.672×10^{-11}	0
9	300	1.041×10^{-10}	0
10	350	1.214×10^{-10}	0
11	400	1.387×10^{-10}	0
12	450	1.561×10^{-10}	0
13	500	1.734×10^{-10}	0

Table 4.5 Result of Static Analysis

After going through the result of the above table, we see that the proving ring is safe for a load of 500N . Though the dynamic analysis is done for a small loads but static analysis shows that the ring is safe for the load of 500N. It may be safe for the higher load also but as per our current requirement the testing up to 500N is enough. Also we see that there is no plastic strain developed in the ring so our working limit is the elastic region only. So the conclusion of the static analysis is that we can use this ring for dynamic analysis to see the effect of various type of loading.

Dynamic Analysis

Before going to the dynamic analysis of the proving ring with piezoelectric layers , the natural frequencies of the model has been calculated by using the step *frequency* and then *model dynamics*. After the calculation of natural frequencies of the system , the analysis has been divided in two parts.

The analysis under the variation of load keeping the frequency constant.

The analysis under the constant load with the variation of frequency.

After the analysis the result are collected in graphical and numerical form and presented. Accordingly this analysis gives us the idea that a proving ring can also be used in calibration of equipments which apply dynamic loading.

The analysis under the variation of load keeping the frequency constant.

In this part of the analysis the analysis is done by keeping the frequency of loading constant at 100 cycles /second. The analysis time is 0.1 second. The output was calculated in the form of Electric flux at the outermost and inner most surfaces of the piezoelectric strips. The load is varied from 5N to 65 N in various steps.

After analyzing minutely, we find that the output we are getting is in the form of sinusoidal curves, we also find that on increasing the load the output flux value is increasing but the response is almost the same wave form . It implies that on increasing the load the out put flux value is affected not the type of out put. The numerical data related to electric flux range with the variation in loading is given in the Table 4.6

S. No.	Load (N)	Frequency (cycles/second)	Range of Electric Flux	
			Inner Layer	Outer Layer
1	5	100	$-15 \times 10^{-12} \text{ --- } 15 \times 10^{-12}$	$-15 \times 10^{-12} \text{ --- } 20 \times 10^{-12}$
2	10	100	$-0.04 \times 10^{-9} \text{ --- } 0.02 \times 10^{-9}$	$-0.02 \times 10^{-9} \text{ --- } 0.04 \times 10^{-9}$
3	15	100	$-0.05 \times 10^{-9} \text{ --- } 0.04 \times 10^{-9}$	$-0.04 \times 10^{-9} \text{ --- } 0.06 \times 10^{-9}$
4	20	100	$-0.06 \times 10^{-9} \text{ --- } 0.05 \times 10^{-9}$	$-0.05 \times 10^{-9} \text{ --- } 0.07 \times 10^{-9}$
5	25	100	$-0.10 \times 10^{-9} \text{ --- } 0.06 \times 10^{-9}$	$-0.06 \times 10^{-9} \text{ --- } 0.10 \times 10^{-9}$
6	30	100	$-0.12 \times 10^{-9} \text{ --- } 0.07 \times 10^{-9}$	$-0.28 \times 10^{-9} \text{ --- } 0.16 \times 10^{-9}$
7	35	100	$-0.13 \times 10^{-9} \text{ --- } 0.08 \times 10^{-9}$	$-0.30 \times 10^{-9} \text{ --- } 0.20 \times 10^{-9}$
8	40	100	$-0.15 \times 10^{-9} \text{ --- } 0.10 \times 10^{-9}$	$-0.35 \times 10^{-9} \text{ --- } 0.25 \times 10^{-9}$
9	45	100	$-0.16 \times 10^{-9} \text{ --- } 0.11 \times 10^{-9}$	$-0.40 \times 10^{-9} \text{ --- } 0.25 \times 10^{-9}$
10	50	100	$-0.18 \times 10^{-9} \text{ --- } 0.12 \times 10^{-9}$	$-0.42 \times 10^{-9} \text{ --- } 0.27 \times 10^{-9}$
11	55	100	$-0.20 \times 10^{-9} \text{ --- } 0.15 \times 10^{-9}$	$-0.50 \times 10^{-9} \text{ --- } 0.30 \times 10^{-9}$
12	60	100	$-0.23 \times 10^{-9} \text{ --- } 0.16 \times 10^{-9}$	$-0.55 \times 10^{-9} \text{ --- } 0.35 \times 10^{-9}$
13	65	100	$-0.25 \times 10^{-9} \text{ --- } 0.20 \times 10^{-9}$	$-0.60 \times 10^{-9} \text{ --- } 0.40 \times 10^{-9}$

Table 4.6 : Electric Flux Range for Load Variation at Constant Frequency

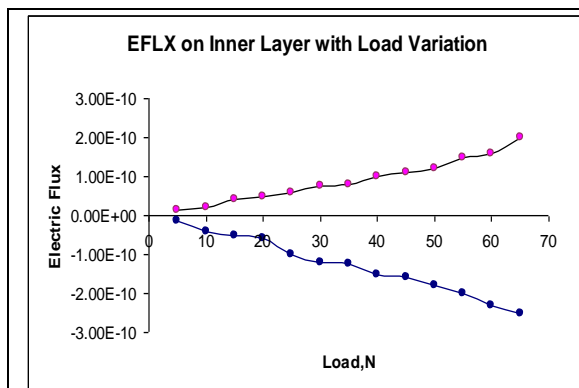


Fig 4.39 Flux on Inner layer with load variation

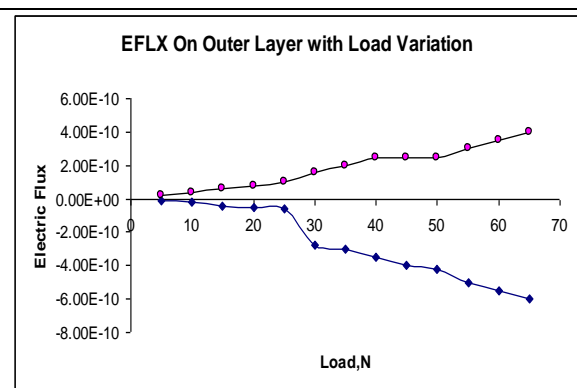


Fig 4.40 Flux on Outer layer with load variation

On the basis of the result given in the table, we find that on increasing the load, the range of electric flux is increasing continuously. If we go for the experimental setup then definitely we will be having the electrical response which will be dependent on the loading magnitude and in this way the calibration of the instrument can be done by using the known applied dynamic load. So such an instrument which was used only for static loading can be converted in more useful form by making it useful for dynamic loading by simply adding the piezoelectric strips to its body.

The analysis under the constant load with the variation of frequency.

In this part of the analysis the analysis is done by keeping the load constant at 5N and frequency is varying. The analysis time is 0.1 second. The output was calculated in the form of Electric flux at the outermost and inner most surfaces of the piezoelectric strips. The frequency is varied from 1000 cycles/sec to 15000 cycles/sec in various steps.

The above analysis is done for the single ring and also for very small loads. For heavy loads the multi ring can be used for the calculation of dynamic loading.

5.2 Response of Piezoelectric Under Varying Temperature

4.2.1 Finite element modeling of cylindrical shell made of PZT-4 and having end cover of Brass and determination of eigen values, mode shapes and calculation of output voltage with the variation of temperature.

Modeling

The modeling of this type shell is done by taking 2-D asymmetric element. The upper half of the diagram as shown in following figure is constructed. And then analysis is done

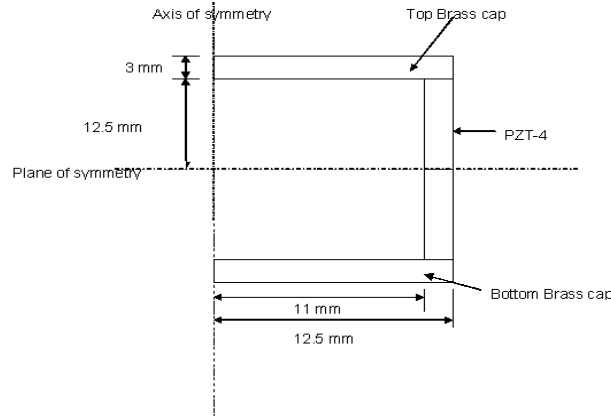


Fig 4.53 Model of piezoelectric shell with Brass end cap

Considering the following:-

Element type

Brass

CAX4R: A 4 node bilinear axisymmetric quadrilateral

PZT-4

CAX8RE: An 8 node biquadrate axisymmetric piezoelectric quadrilateral.

Output

Electric potential on piezoelectric element.

RESULT

On applying the same loading and boundary conditions, following result was obtained in varying temperature conditions-

Temperature(°C)	25	100	150	200	250
Output Voltage	29.58	6.909	-3.92	-22.34	-11.77

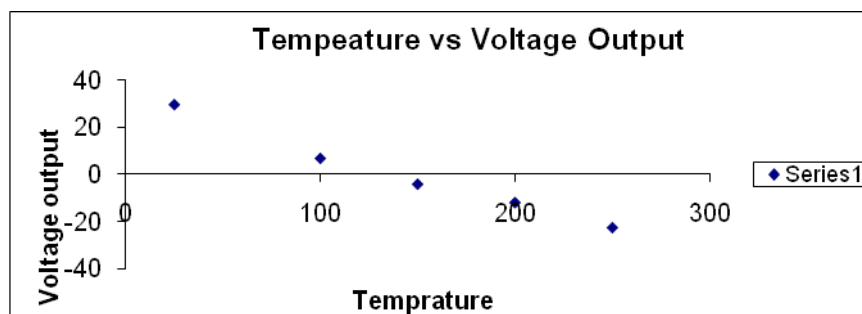


Fig 4.54 Output voltage of Piezoelectric with temperature

5.2.2 Finite element modeling of piezoelectric transducer PZT-4 and determination of its response under periodic loading with the variation in temperature.

Modeling

To solve this problem we consider a 2-D planer cantilever beam fixed at one end. In this cantilever beam we attach a piezoelectric element (PZT-4) as shown in figure. A periodic load is applied on the right most corner with different frequencies.

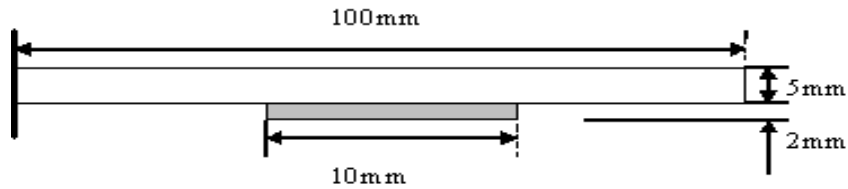


Fig 4.55: Model of beam with piezoelectric element.

Material

Piezoelectric Element - PZT-4

Element

CPS4R – A 4 node bilinear plane stress quadrilateral.

CPE4E - A 4 node bilinear plane strain piezoelectric quadrilateral.

Load

Periodic

Output

Electric potential on piezoelectric element.

Result

On performing the same task , following result was obtained in varying temperature conditions

Temperature(°C)	25	100	150	200	250
Output Voltage	58.43	43.60	33.67	8.887	1.112

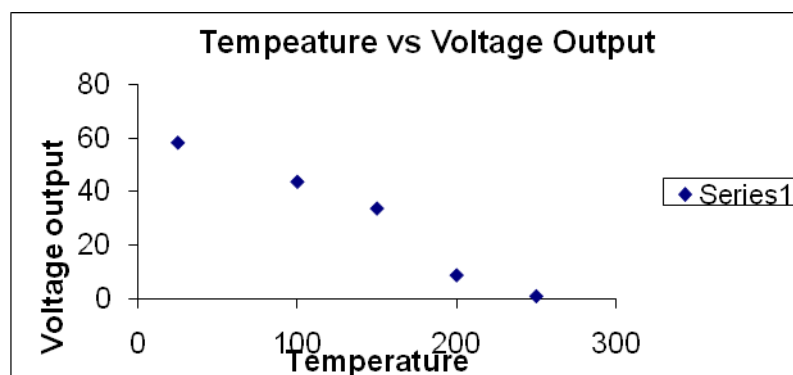


Fig 4.56 Output voltage of Piezoelectric with temperature

After going through the results obtained in section 4.5.1 and 4.5.2, we find that the electrical response of a piezoelectric transducer is dependent on the temperature i.e. working environment. As the temperature increases, keeping all the other variables constant, the electrical response start decreasing. Also as described in the literature review, any piezoelectric crystal is able to work only below the curie temperature of that particular crystal. Hence while choosing the

piezoelectric for a particular application we have to take care of the temperature also and also the curie temperature of that particular crystal.

VI. CONCLUSION

In the studies as described in earlier chapters, the main emphasis is given to the sensor. Piezoelectric behave as sensors when it gives electrical response on the application of mechanical loading. On the basis of the result of the following-

In the section 4.1, the analysis of cantilever beam made of only piezoelectric material is done. The study shows that such a beam can be used as a transducer for the dynamic loading. Natural frequency of the system is also an important part so that we can avoid the loading frequency with the natural frequency.

In section 4.2, analysis of a shell made of PZT-4 with the brass cap is done. In which mode shape has been drawn and also the electrical response has been calculated. The same model has been tested for dynamic loading for a frequency of 250 cycles/sec and for a loading of 20N to 200 N. In which we find that electrical flux output is continuously increasing with increment of load. Thus we can say that such a transducer can be used as dynamic loading.

In section 4.3 analysis of a cantilever beam with a piezoelectric strip as a transducer is done. It is analyzed at a frequency of 250 cycles/ second and a load from 10N to 120N. We observe that with the increase in load the variation in electrical flux output is linear as shown in figure 4.22. Also we can understand that in stead of a complete beam of piezoelectric material, we can use a small strip of piezoelectric as transducer in big structures to analyze dynamic loading.

In section 4.4 the analysis of proving ring with piezoelectric strip by varying load and frequency is done. When we vary the load, we find that there is regular increment in electrical flux output while the frequency remains constant. and when the load is constant at 5N and frequency varied from 1000 cycles/sec to 15000cycles/sec then we find there is no change in the electrical flux output value even though the frequency changes. Hence by using such transducers we can get the magnitude as well as frequency of the dynamic load with proper calibration.

In the section 4.5 the effect of temperature has been observed by which we can say that temperature has a good impact on the response of piezoelectric transducer as shown in Fig 4.54. and 4.56.

In the studies as mentioned above, the main emphasis is given to the sensor. Piezoelectric behave as sensors when it gives electrical response on the application of mechanical loading. After completing the above studies, we can now conclude that a piezoelectric sensor can be modeled successfully by finite element method even without any experimentation. After the detailed study of the work is done, we find that the electrical response of any sensor made of piezoelectric is dependent on the following factors –

Loading condition i.e. what type of loading is there, loading frequency and magnitude of load.

Temperature of the work environment.

Natural frequency of the system.

Curie temperature of the piezoelectric material.

Other then the above mentioned factors the electrical response also depend on the material properties, type of boundary condition and also the type of application.

VII. FUTURE PROSPECT

An important significance of the work presented in the previous sections is to analyze the piezoelectric transducers by finite element analysis and thus actual experiments can be avoided which is a time consuming and costly process. Piezoelectric transducers are widely used for the system where dynamic loading is present such as miniature accele-

rometer, high frequency accelerometer , load cell etc. all such systems can be modeled and analyzed by finite element analysis. Other than sensors, the reverse of it i.e. actuators can also be modeled analyzed by the finite element method. We can also develop some systems on FEA packages and can understand the effect of various type of geometries , shape , size etc on the response of transducer. Also the temperature effect can be analyzed more elaborately .

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