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PARAMETERIZED FILTER DESIGN OF BIORTHOGONAL WAVELET FOR EDGE FEATURE EXTRACTION

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Abstract

Edge feature extraction is basic and important subject in computer vision, medicine and image processing. Edge feature extraction of biorthogonal wavelet is more preferable when compared with orthogonal wavelets because of more flexibility and less computation time. There are different methods for the design of biorthogonal wavelet i.e. Multi Resolution Analysis (MRA) and Lifting Wavelet transform but their realization is limited and difficult. This Paper proposes a parameterized design of biorthogonal wavelet based on the algebraical construction method. In order to assign the characters of biorthogonal wavelet, two kinds of parameters are imported in construction process. One is scale factor and another one is sign factor. By choosing different sign, the waveform shape can be changed distinctly. Using scale factor, the detail such as softness can be adjusted. The biorthogonal wavelet design can be adjusted by parameters. So the edge position is more accuracy in multi scale image edge detection.

Keywords: Biorthogonal Wavelet, Parameterized, Edge Detection, Symmetry, Wavelet Construction.

I. INTRODUCTION

Edge is the most the important information of image. Using wavelet transform to detect image edge is popular method. The Wavelet representation for an image corresponds to a decomposition of the image into a set of independent frequency bands: approximation and three spatial: horizontal and vertical and diagonal orientations. Discrete wavelet transform is very useful in edge detection [3]. For the properties of varied wavelet has many differentia, such as orthogonality and symmetry and vanishing moments, the qualities of detected edge are different. So the characteristics of biorthogonal wavelet are essential to improving edge detection results.

Now there have two construction methods of biorthogonal wavelet. One is Multi-Resolution Analysis (MRA) and another is Lifting wavelet transform. Though the two methods can construct or improve biorthogonal wavelet, the realization is difficult and limited. In this paper, the parameterized filter design of biorthogonal wavelet is proposed. By choosing different characteristics, a serial of biorthogonal wavelets can be constructed by parameterized filter design. And the detail of biorthogonal wavelet can be adjusted by parameters.

Wavelet modulus maximum algorithm is generated based on multi-scale decomposition. The algorithm is composed of following steps [1].

1) Count the wavelet transform coefficients on two directions in multi-scale. Include the detail coefficients and the approximate coefficients;
2) Find the modulus maximum point by comparing the neighbour points;
3) Deleting the points whose modulus is smaller than threshold;
4) Combining the edge results in multi-scale;

In wavelet modulus maximum algorithm, the selection of threshold is difficult. If the threshold is larger than ideal value, some details of edge may be deleted. Conversely, If the threshold is too small, a lots of pseudo edge may be preserved and the real edge information may be covered.

II. BIORTHOGONAL WAVELET THEORY

The design of orthonormal wavelets requires a step known as spectral factorization, which can make the filter lengths grow when going to coarser scales; moreover, orthogonal filters cannot be symmetric. These limitations are also encountered in classical wavelet filter design, and they can be circumvented by relaxing the orthogonality condition and considering biorthogonal wavelet.

Daubechies said that the only symmetric, finite length, orthogonal filter is the haar filter [6]. While talking about the limitations of the haar wavelet, the shorter filter length sometimes fails to detect large changes in the input data. So we are interested to design symmetric filters of length greater than 2[9]. The goal is to construct two low pass filters h and $\tilde{h}$ and their associated high pass filters g and $\tilde{g}$.

2.1 Scaling and Wavelet functions

Let $(h_n)_{n\in Z}$ and $(\tilde{h}_n)_{n\in Z}$ be the finite real sequences, the associated scaling functions ϕ and $\tilde{\phi}$ are recursively defined as

$$\phi(x) = \sqrt{2}\sum_n h(n)\phi(2x-n),$$

and $$\tilde{\phi}(x) = \sqrt{2}\sum_n \tilde{h}(n)\tilde{\phi}(2x-n) \qquad (1)$$

The associated wavelets ψ and $\tilde{\psi}$ are defined as,

$$\psi(x) = \sqrt{2}\sum_n g_{n+1}\phi(2x-n),$$

and $$\tilde{\psi}(x) = \sqrt{2}\sum_n \tilde{g}_{n+1}\tilde{\phi}(2x-n) \qquad (2)$$

Where $g_{n+1} = (-1)^n \tilde{h}(1-n)$, and $\tilde{g}_{n+1} = (-1)^n h(1-n)$.Such a set of four functions $\{\phi, \tilde{\phi}, \psi, \tilde{\psi}\}$forms a two band biorthogonal wavelet system [10].

2.2 Symmetric filters

It is possible to construct smooth biorthogonal wavelets of compact support that are either symmetric or antisymmetric. This is impossible for orthogonal wavelets, besides particular case of the Haar basis. Symmetric or antisymmetric wavelets are synthesized with perfect reconstruction filters having a linear phase.

Consider all filters are finite length. If the filter length is odd, we require our filter to be symmetric about zero and when the filter length is even, filter be symmetric about 1/2 [4][5].

Let h= $(h_l......h_L)$ be a finite length filter with length N=L-l+1. We say that h is symmetric if

(a) $h_k = h_{-k}$ for all k $\in$ Z whenever N is odd.

(b) $h_k = h_{1-k}$ for all k $\in$ Z whenever N is even.

Symmetric filters are good for minimizing the edge effects in the representation of the discrete wavelet transform (DWT) of a function [7][2].Larger coefficients results the false edges due to periodization is avoided.

2.3 Orthogonality in the Fourier Domain

The Orthogonality conditions that the filter pairs h and $\tilde{h}$, g and $\tilde{g}$ must satisfy

$$\tilde{H}(\omega)\overline{H(\omega)} + \tilde{H}(\omega+\pi)\overline{H(\omega+\pi)} = 2 \qquad (3)$$

If $H(\omega)$ and $\tilde{H}(\omega)$ satisfy, then we have

$$\sum_{k\in Z} \tilde{h}_k h_k = 1 \qquad (4)$$

and for $m \in Z, m \neq 0,$

$$\sum_{k\in Z} \tilde{h}_k h_{k-2m} = 0 \qquad (5)$$

III. PARAMETERIZED FILTER DESIGN

The Filter design of biorthogonal wavelet is based on low pass filter relation and perfect reconstruction condition. Then the expression of low pass decomposition filter and high pass reconstruction filter can be solved. When using wavelet transforms to deal image, the first step is decompose the image. The high pass detail information is drawn out[8]. The edge can be obtained by taking high pass coefficient and reconstruction. The process shows that high pass decomposition filter is place predominant role than other filter in wavelet transform. The sign sequence of filters element gives effect to the capability of edge detection.

The step of using Multi Resolution Analysis to construct the biorthogonal wavelet is:

$$\tilde{h}, h \Rightarrow \tilde{g}, g \quad \tilde{\phi}, \phi \Rightarrow \tilde{\psi}, \psi$$

In order to gain the expected characteristics of high pass decomposition filter, the process of construction is updated.

$$\tilde{\psi} \rightarrow \phi \rightarrow \tilde{\phi} \rightarrow \psi \qquad \tilde{g} \rightarrow h \rightarrow \tilde{h} \rightarrow g$$

In this case, $\tilde{g}$ and $\tilde{h}$ is the high pass and low pass decomposition filter, g and h is the high pass and low pass reconstruction filter. The parameterized filter design approach for biorthogonal wavelet is shown in Figure 1.

There are four steps in parameterized filter design of biorthogonal wavelet. The first step is to select the characteristics of high pass decomposition filter $\{\tilde{g}\}$, such as symmetry. The second step is parameterized design of high pass decomposition filter. The scale factor [k] and sign factor '±'is added in filter vector. If the support interval of filter is too long, the scale factor is [k_1, k_2, k_3.....k_n]are used. So the proportion of adjacent filter elements can be adjusted independent by scale factors. The positive sign or the negative sign impacts the filters elements sign sequence. Third, the parameterized low pass reconstruction filter $\tilde{g}$ through the following equation.

$$h_k = (-1)^{k-1}\tilde{g}_{1-k} \quad (6)$$

In the last step, using perfect reconstruction condition (PR) and biorthogonal filter condition and vanishing moments, the parameterized expression of low pass decomposition filter $\tilde{h}$ and high pass reconstruction filter g can be obtained.

Biorthogonal filter condition is

$$\begin{cases} \sum_k h_{2k} = \sum_k h_{2k+1} = 1/\sqrt{2} \\ \sum_k \tilde{h}_{2k} = \sum_k \tilde{h}_{2k+1} = 1/\sqrt{2} \end{cases} \quad (7)$$

Perfect reconstruction condition is[1]

$$\begin{cases} \sum_k h_k \tilde{h}_{k-2n} = \delta_{0,n} \\ \tilde{g}_n = (-1)^n h_{1-n} \\ g_n = (-1)^n \tilde{h}_{1-n} \end{cases} \quad (8)$$

3.1 Odd Support Interval

Assuming the Support interval of biorthogonal wavelet low pass filter h and $\tilde{h}$ are odd. The length are N_1 and N_2 respectively. Because the low pass filter of wavelet is even symmetry, then

$$h = (h_{\frac{1-N_1}{2}}, \ldots\ldots, h_0, \ldots\ldots, h_{\frac{N_1-1}{2}})$$

$$\tilde{h} = (\tilde{h}_{\frac{1-N_2}{2}}, \ldots\ldots, \tilde{h}_0, \ldots\ldots, h_{\frac{N_2-1}{2}})$$

The biorthogonal wavelet filter must fulfill the condition

$$\sum_k h_k h_{k-2n} = \delta_{0,n} \qquad n \in Z$$

3.2 Even Support Interval

If the support interval of low pass reconstruction filter h is even, we defined the length as N . The expression of h is

$$h = (h_{-\frac{N}{2}}, \ldots\ldots, h_0, h_1, \ldots\ldots\ldots h_{\frac{N}{2}})$$

By (1), the expression of $\tilde{g}$ is

$$\tilde{g} = (\tilde{g}_{1-\frac{N}{2}}, \ldots\ldots\ldots, \tilde{g}_0, \tilde{g}_1, \ldots\ldots\ldots\ldots \tilde{g}_{1+\frac{N}{2}})$$

$$=((-1)^{1-\frac{N}{2}}h_{\frac{N}{2}},........,h_1,-h_0,h_{-1},............(-1)^{1+\frac{N}{2}}h_{-\frac{N}{2}}) \qquad (9)$$

For the low pass filter is even symmetry, then

$$h=(h_{\frac{N}{2}},.......,h_1,h_1,........h_{\frac{N}{2}}) \qquad (10)$$

and $\tilde{g}=((-1)^{1-\frac{N}{2}}h_{\frac{N}{2}},........,h_1,-h_1,............(-1)^{\frac{N}{2}}h_{\frac{N}{2}})$ (11)

Clearly, the high pass decomposition filter $\tilde{g}$ is odd symmetry based on 1/2 point when the support interval of h is even. If we need to design the filter $\tilde{g}$ with even symmetry, the support interval of $\tilde{g}$ must be odd.

IV. WAVELET CONSTRUCTION IN EDGE DETECTION BY PARAMETERIZED FILTER DESIGN

Parameterized algebraical construction method can be used to design biorthogonal wavelet with even support interval. We assuming the support interval of high pass decomposition filter $\tilde{g}$ is six. The expression of $\tilde{g}$ and h is

$$\tilde{g}=\{\tilde{g}_{-2},\tilde{g}_{-1},\tilde{g}_0,\tilde{g}_1,\tilde{g}_2,\tilde{g}_3\}$$
$$h=\{h_{-2},h_{-1},h_0,h_1,h_2,h_3,\}$$

The sign of high pass decomposition filter monotonic, the relations between high pass filter elements is set as

$$\tilde{g}_0=-\tilde{g}_1,\quad \tilde{g}_{-1}=-\tilde{g}_2,\quad \tilde{g}_{-2}=-\tilde{g}_3$$

And the two scale factors $\{k_1,k_2\}$ and $k_1>0$ $k_i\neq 1$ $i=1,2$. Defining

$$\tilde{g}_1=k_1\tilde{g}_3 \qquad \tilde{g}_2=k_2\tilde{g}_3$$

Then, the high pass filter is expressed as

$$\tilde{g}=\{-\tilde{g}_3,-k_2\tilde{g}_3,-k_1\tilde{g}_3,k_1\tilde{g}_3,k_2\tilde{g}_3,\tilde{g}_3\}$$

Form (6), the low pass reconstruction filter is

$$h=\{-\tilde{g}_3,k_2\tilde{g}_3,-k_1\tilde{g}_3,-k_1\tilde{g}_3,k_2\tilde{g}_3,-\tilde{g}_3\}$$

If defining the Support interval of low pass decomposition filter is six too. Then

$$\tilde{h}=\{\tilde{h}_{-2},\tilde{h}_{-1},\tilde{h}_0,\tilde{h}_1,\tilde{h}_2,\tilde{h}_3\}\ \tilde{h}=\{\tilde{h}_3,\tilde{h}_2,\tilde{h}_1,\tilde{h}_1,\tilde{h}_2,\tilde{h}_3\}$$

Form filter condition (7), we can get

$$-\tilde{g}_3+k_2\tilde{g}_3-k_1\tilde{g}_3=\sqrt{2}/2$$

From Perfect reconstruction condition (8), we can get

$$2\left(-\tilde{h}_3\tilde{g}_3+k_2\tilde{g}_3\tilde{h}_2-k_1\tilde{g}_3\tilde{h}_1\right)=1$$

$$-\tilde{g}_3\tilde{h}_1+k_2\tilde{g}_3\tilde{h}_1-k_1\tilde{g}_3\tilde{h}_1-k_1\tilde{g}_3\tilde{h}_3=0$$

The low pass reconstruction filter be scaled then

$$\tilde{h}_2 = k_2\tilde{h}_3 \qquad (12)$$

The equations which got from vanishing moment condition is

$$\left.\begin{aligned}\tilde{h}_1 &= 3\tilde{h}_2 - 5\tilde{h}_3\\ \tilde{h}_1 &= 9\tilde{h}_2 - 35\tilde{h}_3\end{aligned}\right\} \qquad (13)$$

The system of Linear equations i.e., (9) ,(14), we got the filter coefficients

$$\tilde{h}_1 = 0.441875, \tilde{h}_2 = 0.2209375, \tilde{h}_3 = 0.0441875$$

Substituting the above values in (13) and then we can get the scale factor k_2.then substitute all values in (12) and (11) we can get k_1 and $\tilde{g}_3$.

$$k_1 = \frac{20}{3}, k_2 = 5, \tilde{g}_3 = -0.2651$$

The filters of biorthogonal wavelet is

$$\begin{aligned}
\tilde{h} &= \{0.0441875,\ 0.2209375,\ 0.441875,\ 0.441875,\\ &\quad 0.2209375, 0.0441875\}\\
\tilde{g} &= \left\{\frac{3\sqrt{2}}{16}, \frac{15\sqrt{2}}{16}, \frac{5\sqrt{2}}{4}, -\frac{5\sqrt{2}}{4}, -\frac{15\sqrt{2}}{16}, -\frac{3\sqrt{2}}{16}\right\}\\
h &= \left\{\frac{3\sqrt{2}}{16}, -\frac{15\sqrt{2}}{16}, \frac{5\sqrt{2}}{4}, \frac{5\sqrt{2}}{4}, -\frac{15\sqrt{2}}{16}, \frac{3\sqrt{2}}{16}\right\}\\
g &= \left\{\begin{matrix}0.0441875,\ -0.2209375\ \ 0.441875,\ -0.441875,\\ 0.2209375,\ -0.0441875\end{matrix}\right\}
\end{aligned} \qquad (14)$$

The biorthogonal wavelet named as 'zbo6.6'.where 'zbo' is the wavelet name and '6.6' indicates the number of reconstruction filter coefficients are six and number of decomposition filter coefficients are six.

Figure 2(a) and 2(b) shows the waveform of high decomposition filter and low pass decomposition filter coefficients of 'zbo6.6'.

If the vanishing moments is high, the support interval of high pass decomposition filter $\tilde{h}$ must be longer than 'zbo6.6'. In there, we keep the filter $\tilde{g}$ and h constantly and select the support interval of filter $\tilde{h}$ as ten. The filter $\tilde{h}$ is even symmetry.

$$\begin{aligned}\tilde{h} &= \{h_{-4}, h_{-3}, h_{-2}, h_{-1}, h_0, h_1, h_2, h_3, h_4, h_5\}\\ &= \{h_5, h_4, h_3, h_2, h_1, h_1, h_2, h_3, h_4, h_5\}\end{aligned}$$

The process of construction is same as above. The filter condition and the perfect reconstruction condition is

$$\left.\begin{aligned}
-\tilde{g}_3 + k_2\tilde{g}_3 - k_1\tilde{g}_3 &= \sqrt{2}/2\\
\tilde{h}_5 + \tilde{h}_4 + \tilde{h}_3 + \tilde{h}_2 + \tilde{h}_1 &= \sqrt{2}/2\\
\tilde{h}_4 &= k_2\tilde{h}_5\\
2\left(-\tilde{h}_3\tilde{g}_3 + k_2\tilde{g}_3\tilde{h}_2 - k_1\tilde{g}_3\tilde{h}_1\right) &= 1
\end{aligned}\right\} \qquad (15)$$

$$-\tilde{g}_3\tilde{h}_2 + k_2\tilde{g}_3\tilde{h}_3 - k_1\tilde{g}_3\tilde{h}_4 - k_1\tilde{g}_3\tilde{h}_5 = 0 \quad \tilde{g}_3\tilde{h}_1 - k_2\tilde{g}_3\tilde{h}_1 + k_1\tilde{g}_3\tilde{h}_3 - k_2\tilde{g}_3\tilde{h}_4 + \tilde{g}_3\tilde{h}_5 = 0$$

In this case, the equations from vanishing moments is

$$\left.\begin{aligned} \tilde{h}_1 &= 3\tilde{h}_2 - 5\tilde{h}_3 + 7\tilde{h}_4 - 9\tilde{h}_5 \\ \tilde{h}_1 &= 9\tilde{h}_2 - 35\tilde{h}_3 + 91\tilde{h}_4 - 189\tilde{h}_5 \\ \tilde{h}_1 &= 15\tilde{h}_2 - 65\tilde{h}_3 + 175\tilde{h}_4 - 369\tilde{h}_5 \\ \tilde{h}_1 &= 33\tilde{h}_2 - 275\tilde{h}_3 + 126\tilde{h}_4 - 4149\tilde{h}_5 \end{aligned}\right\} \tag{16}$$

The system of Linear equations i.e., (15),(16), we got the filter coefficients

$$\left.\begin{aligned} &h_1 = 0.1547, h_2 = 0.2320, \\ &h_3 = 0.2099, h_4 = 0.0939, h_5 = 0.0166 \\ &k_1 = 8.6667, k_2 = 5.6667, g_3 = -0.1768 \end{aligned}\right\} \tag{17}$$

The filter coefficients of biorthogonal wavelet 'zbo6.10'is

$$\left.\begin{aligned} &\tilde{h} = \{0.0166, 0.0939, 0.2099, 0.2320, 0.1547, \\ &\quad 0.1547, 0.2320, 0.2099, 0.0939, 0.0166\} \\ &\tilde{g} = \{0.1768, 1.0017, 1.5321, -1.5321, -1.0017, -0.1768\} \\ &h = \{0.1768, -1.0017, 1.5321, 1.5321, -1.0017, 0.1768\} \\ &g = \{0.0166, -0.0939, 0.2099, -0.2320, 0.1547, \\ &\quad -0.1547, 0.2320, -0.2099, 0.0939, -0.0166\} \end{aligned}\right\} \tag{18}$$

The wavelet named as 'zbo6.10', '6.10'indicates the number of reconstruction filter coefficients is 6 and number of decomposition filter coefficients is 10.

Figure 3(a) and 3(b) shows the waveform of high pass decomposition filter and low pass decomposition filter coefficients of 'zbo6.10'. By observing the Figure 2 and Figure 3, the 'zbo6.6' is the most smoothness of two wavelets.

V. SIMULATION RESULTS

In order to test the performance of these wavelets, we use ribs.jpg image shown in Figure 3. All test realized in MATLAB. In testing, one level decomposition is done, the range of threshold values are 0.03 and 0.06. And the threshold of each scale counted by adaptive threshold algorithm. The Tabel 1 shows the comparision of two biorthogonal wavelets with respect to computation time and edgepoints.

The Figure 4 shows the Edge results of Parametrized filter design of biorthogonal wavelet i.e., 'zbo6.6'& 'zbo6.10'.By analyzing the Fig 4, the main difference reflects at two aspects between two wavelets.

1) Visual effects:From the figure 4. it is clear that 'zbo6.6' is better than other wavelet 'zbo6.10'. The main features of Original image is extracted efficeintly. The discussion of visual effects can be stated that the monotonic sign is the most important property of high pass decompostion filter in edge detection.

2) Computational time: From Table 1 shows that, we can get faster response from the wavelet 'zbo6.10' than 'zbo6.6'.

3) edge points : edge points represents the strength of the image. The maximum edge points is obtained for 'zbo6.10'. The detected edge points are 21465, but its edges are not visibly clear. The wavelet 'zbo6.6' result got fine edges with detected edge points are 13359. This means there are many pseudo edge points in figure 4(b).

VI. CONCLUSION

In this paper, we have proposed a new method of design biorthogonal wavelet. In process of designing wavelet, we import two kind parameters. One is sign, which is used to change the main characters of wavelet, such as waveform shape. Another is scale factor, which is used to adjust waveform in detail. If sign is same, we can improve the convergence of low pass filter by increasing the factor. The process of construction is simply and flexible.

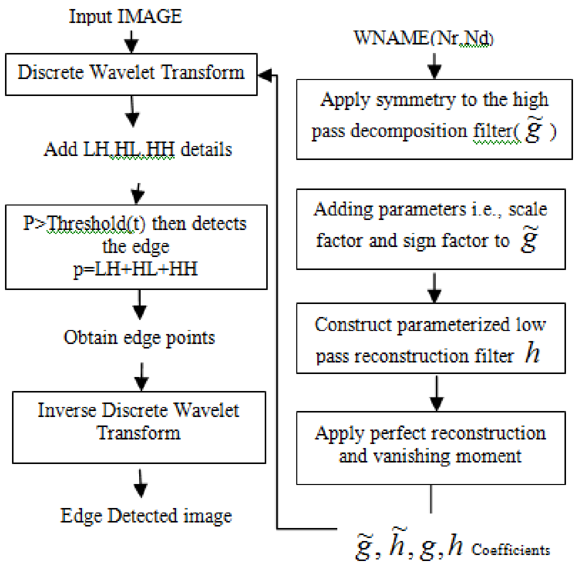

Figure 1: Block diagram of parameterized filter design of Biorthogonal wavelet

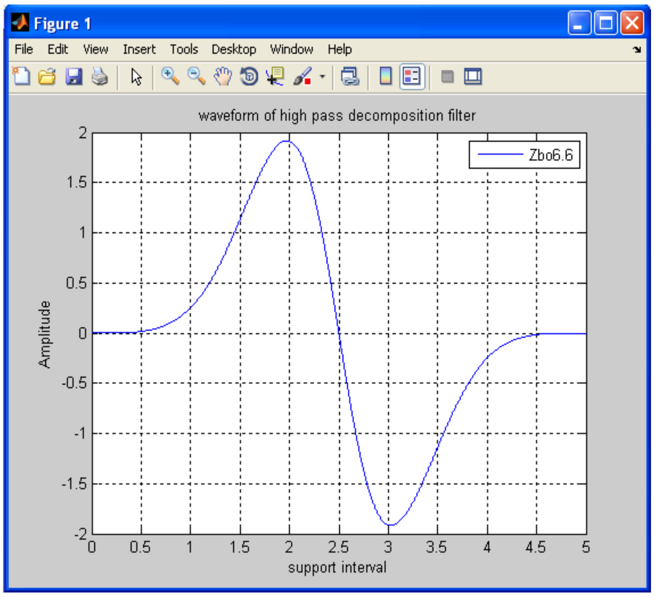

Figure:2(a)

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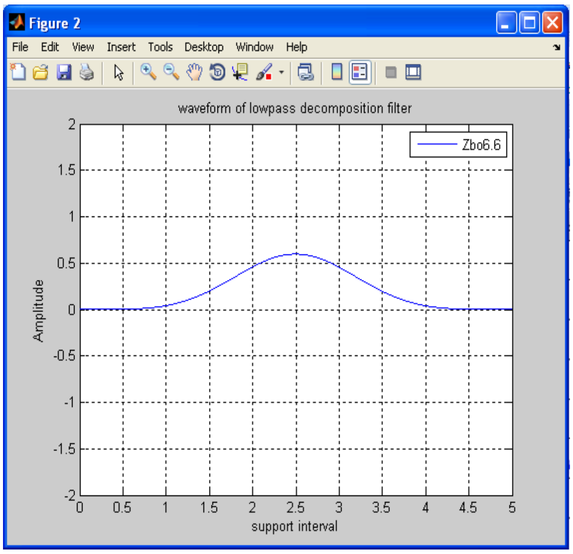

Figure :2(b)

Figure 2: (a) high pass and (b) low pass decompostion filters of 'zbo6.6'.

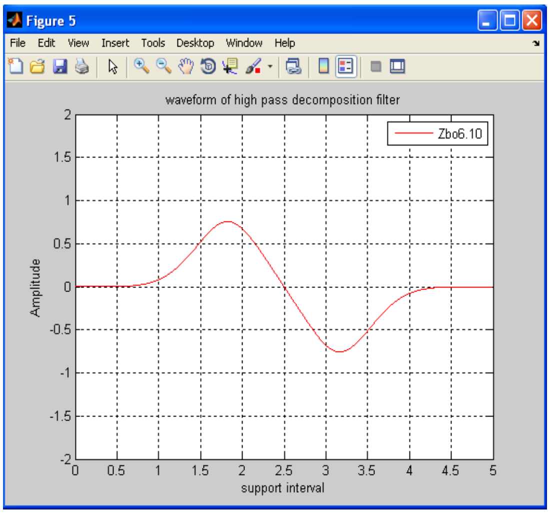

Figure:3(a)

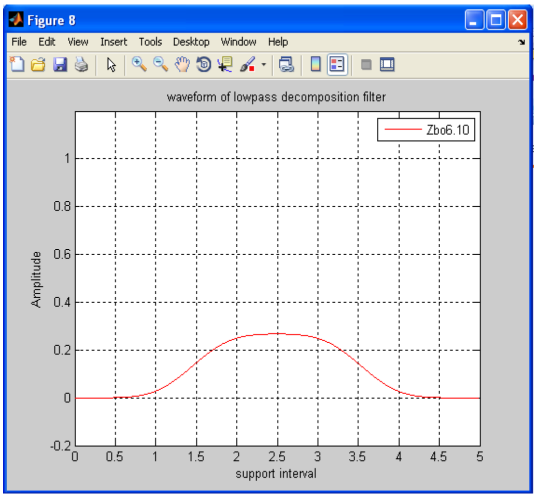

Figure :3 (b)

Figure 3: (a) High pass and(b) Lowpass decompostion filters of 'zbo6.10'

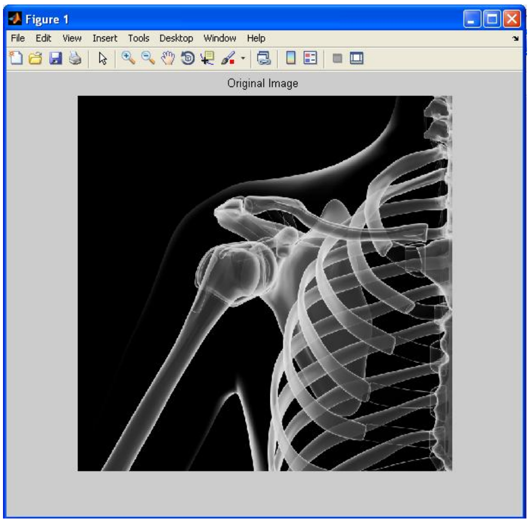

Figure :4(a) Original Image

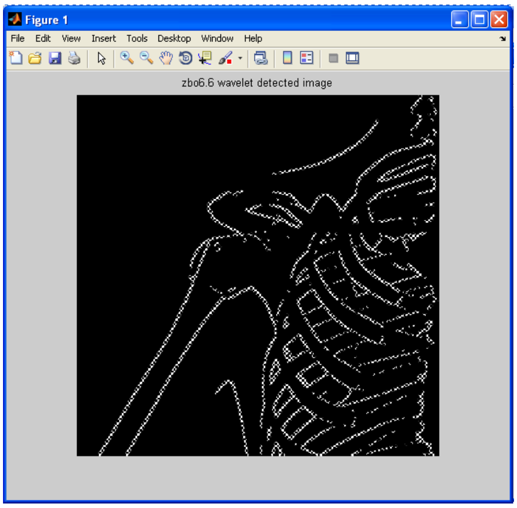

Figure:4 (b) Edge detected image by 'zbo6.6'wavelet

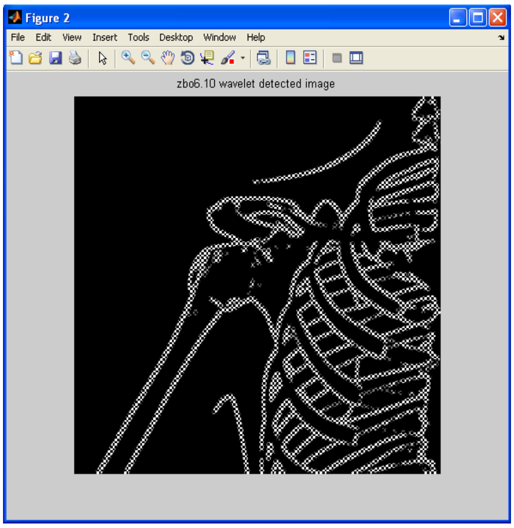

Figure :4(c) Edge detected image by 'zbo6.10' wavelet

Figure 4. Edge results of Parametrized filter design of biorthogonal wavelets 'zbo6.6' and 'zbo6.10'

Table :1 Comparisions of Biorthogonal Wavelets

avelet type	Computation time in sec	edgepoints
zbo6.6	2.4063	13359
zbo6.10	1.9063	21465

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