

OPTIMUM COEFFICIENT OF PERFORMANCE OF IRREVERSIBLE CARNOT MAGNETIC REFRIGERATOR

Aaditya Mishra¹, Govind Maheshwari²

¹Department of Mechanical Engineering, Institute of engineering & Technology,
Devi Ahilya University, Indore, (India)

²Department of Mechanical Engineering, Institute of engineering & Technology,
Devi Ahilya University, Indore, (India)

ABSTRACT

Considering the multi irreversibilities, an irreversible Carnot Cycle model is established. For which factors like finite time analysis, heat leakages are included to find out the expressions for several important performance parameters, such as the cooling rate, coefficient of performance (COP). COP has been optimized in terms of highest temperature. For the optimum region the COP and Refrigeration Effect has been evaluated. Various performance curves for highest temperature range, COP and Refrigeration Effect has been drawn. Results obtained are giving the generalized performance behavior of magnetic refrigerator in terms of COP.

Keywords: Paramagnetic Salt, Carnot Refrigerator, Irreversibility, Finite Time Analysis, Ecological Function

I INTRODUCTION

Magnetic refrigeration is technology based on magneto caloric effect (MCE). Warburg [1] observed MCE. Magneto-caloric effect is the reason behind the increase in temperature when magnetized and decrease in temperature when demagnetized. This MCE is valid for all the magneto-caloric materials. Weiss and Piccard [2] had also given the explanations of Magneto-caloric effect. Later Debye [3] and Giauque [4] proposed a method of magnetic refrigeration for low temperature physics in order to obtained sub-kelvin temperatures. Verification of the method to achieve the sub-kelvin temperature was done by Giauque and MacDougall[5], experimentally. Since then “Magnetic Refrigeration” was a standard technique in low temperature refrigeration. After knowing the hazardous effects of cfc refrigerants in vapour compression refrigeration which is highly efficient , it became the requirement to indulge some other technique in the field of refrigeration, in a good response to cop conferences. There were number of good alternatives like Magneto-caloric refrigeration, Thermo-elastic refrigeration, Elasto-caloric

refrigeration. Where magneto-caloric refrigeration utilizes Magneto-caloric effect. Magnetocaloric refrigeration is used with the help of magnetocaloric materials like Gadolinium, Magnise and there compounds with them. Curie point temperature is the main culprit in these materials, which is very low in room temperature conditions. $Gd_5(Si_xGe_y)$, $GdZn$, $Tb_xY_{1-x}Fe_2$ etc[6] are commonly used refrigerents. Still so much research is demanded in the field of Magneto-caloric materials because of low range of working temperature due to which COP and refrigeration effect are quit low in this kind of refrigerator. Thermodynamic analysis [7]of magnetic refrigeration suggest the need to optimize the cyclic process in the real world situation. As the temperature range at room temperature is too low for magnetic refrigeration so cascade system and system with regeneration is also very good way to improve performance in the magnetic refrigerator.

II AN IRREVERSIBLE MODEL OF MAGNETIC CARNOT REFRIGERATION CYCLE

An irreversible Carnot refrigeration cycle with paramagnetic material (magnetic intensity is not very high or temperature is very low) is shown schematically in fig. 1.1 This is a well defined fact that Carnot cycle is a combination of two isothermal processes and two adiabatic processes. Where in practical situation heat absorption and heat rejection at isothermal processes are infinitely slow and any adiabatic process are always infinitely fast. It will get infinite time to get a finite amount of refrigeration effect. So , to get a finite amount of refrigeration in finite time the cycle should be speeded up. However to get a heat flux in isothermal processes a temperature difference will be added. So adding a temperature difference

At higher and lower temperature side. Allotting T_H and T_C as hot and cold reservoir temperatures and T_2 and T_1 are hot and cold working substance temperatures as shown in “fig 1” below-

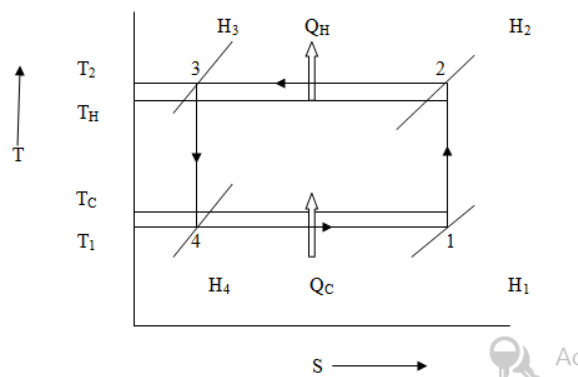


Figure1 T-S Diagram of a Irreversible Magnetic Carnot Refrigeration Cycle

Where

$H_1, H_2, H_3,$ and H_4 are four different magnetic fields,

Q_C and Q_H are the heat absorbed and heat rejected.

Q_{1-4} and Q_{2-3} are the heat transferred from isothermal processes.

Q_{1-2} and Q_{3-4} are zero as the process is adiabatic processes..

$$Q_{2-3} = T_2 (S_2 - S_3) = C \mu_0 (H_2^2 - H_3^2) / 2T_2 = \Delta S / T_2 \quad [14] \quad (1)$$

$$Q_{1-2} = 0 \quad (2)$$

Where

$$\Delta S = C \mu_0 (H_2^2 - H_3^2) / 2$$

μ_0 = vacuum magnetic permeability

$$Q_{1-4} = T_1 (S_1 - S_4) = C \mu_0 (H_1^2 - H_4^2) / 2T_1 = \Delta S / T_1 \quad [9] \quad (3)$$

$$Q_{3-4} = 0 \quad (4)$$

C = curie constant = $N g^2 \mu_B J (J+1) / 3K_B$

$$\Delta S = C \mu_0 (H_2^2 - H_3^2) / 2$$

Heat transfer in finite time t_1 and t_2

$$Q_{2-3} = K_2 (T_2 - T_H) t_2 \quad (5)$$

$$Q_{1-4} = K_1 (T_C - T_1) t_1 \quad [8] \quad (6)$$

K = material constant

t = time

Including heat leakage also here

$$Q_L = \text{heat leakage per cycle} = K_L (T_H - T_C) \quad [8] \quad (7)$$

$$= t_1 + t_2 \quad (\text{total time})$$

From equation no (5) and (6)

$$\tau = [Q_{2-3} / K_2 (T_2 - T_H)] + [Q_{1-4} / K_1 (T_C - T_1)] \quad (8)$$

Q_C = heat absorbed

$$= Q_{1-4} - Q_L$$

$$= (\Delta S / T_1) - K_L (T_H - T_C) \tau$$

$$= (\Delta S / T_1) - \Delta S K_L (T_H - T_C) [\{ 1 / K_2 (T_2 - T_H) \} + 1 / K_1 (T_C - T_1)] \quad (9)$$

Q_H = heat rejected

$$= Q_{2-3} - Q_L$$

$$= (\Delta S / T_2) - \Delta S K_L (T_H - T_C) [\{ 1 / K_2 (T_2 - T_H) \} + 1 / K_1 (T_C - T_1)] \quad (10)$$

$$W_{\text{input}} = Q_H - Q_C$$

From eq. (9) and (10)

$$W_{\text{input}} = \Delta S (1/T_2 - 1/T_1) \quad (11)$$

Refrigeration rate

$$R = Q_C / \tau$$

$$= (\Delta S / T_1) - \Delta S K_L (T_H - T_C) [\{ 1 / K_2 (T_2 - T_H) \} + 1 / K_1 (T_C - T_1)]$$

$$\Delta S [\{ 1 / K_2 T_2 (T_2 - T_H) \} + \{ 1 / K_1 T_1 (T_C - T_1) \}]$$

(12)

$$\begin{aligned} \text{COP} &= Q_C / W_{\text{input}} \\ &= (\Delta S / T_1) - \Delta S K_L (T_H - T_C) [\{ 1 / K_2 (T_2 - T_H) \} + 1 / K_1 (T_C - T_1)] \end{aligned}$$

$$\Delta S (1/T_2 - 1/T_1)$$

(13)

III OPTIMIZATION OF COP

As the expression for COP is given in equation no (13)

$$\text{COP} = \frac{(\Delta S / T_1) - \Delta S K_L (T_H - T_C) [\{ 1 / K_2 (T_2 - T_H) \} + 1 / K_1 (T_C - T_1)]}{\Delta S (1/T_2 - 1/T_1)}$$

$$\Delta S (1/T_2 - 1/T_1)$$

According to second law of thermodynamics temperature scales are arbitrary for a reversible cycle, by which

$$Q_{2-3}' / Q_{1-4} = T_2 / T_1 \quad (14)$$

Where

Q_{2-3}' = heat transfer if the process is reversible

But our complete analysis is based on irreversibility, by which for the designed TR or same refrigeration, work input for an irreversible refrigerator will be more than the reversible one. The same thing will happen with heat rejected (Q_{2-3}). As Q_{2-3}' is the heat rejected for the reversible cycle then

$$Q_{2-3}' / Q_{2-3} < 1 \quad (15)$$

$$\text{Let } Q_{2-3}' / Q_{2-3} = \phi \quad (16)$$

By eq. (15) and (16)

$$\phi < 1$$

$$\phi = \text{close to 1, i.e. 0.88}$$

By eq. no. (14) and (16)

$$(\phi Q_H / Q_C) = (T_2 / T_1) \quad (17)$$

Taking

$$(T_1 / T_2) = x = \text{constant}$$

$$\left(\frac{d\text{COP}}{dT_2} \right)_{x=\text{Constant}} = 0$$

Resulting in to the

$$T_2 = A T_C + T_H$$

$$1 + Ax$$

$$\text{Where } A = \sqrt{(T_H / T_C)(K_1 / K_2)}$$

IV PERFORMANCE BASED ON THE OPTIMIZED COP OF THE REFRIGERATOR

Taking

$$T_C = 260 \text{ K}, T_H = 300 \text{ K}, T_o = 290 \text{ K}, K_1, K_2 = 1 \times 10^{-12}, K_L = 0.5 \times 10^{-15}, \gamma = 3 \times 10^{-14}$$

$$H_1 = 2T, H_2 = 4T, H_3 = 8T, H_4 = 6T \text{ [9]}$$

After putting range of temperature values around optimum value of temperature, so many values of COP, Refrigeration Effect can be found. Various performance curves are as follows-

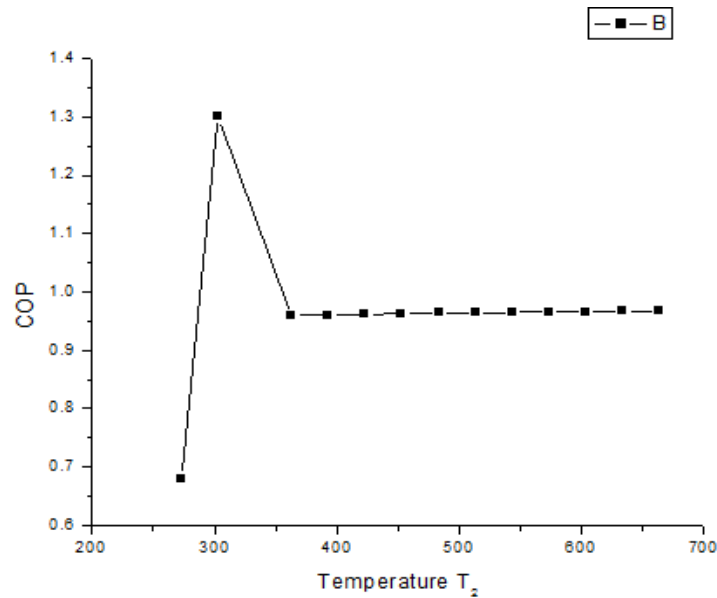


Figure 1 Variation of nondimensional term COP and temperature T_2 (K)

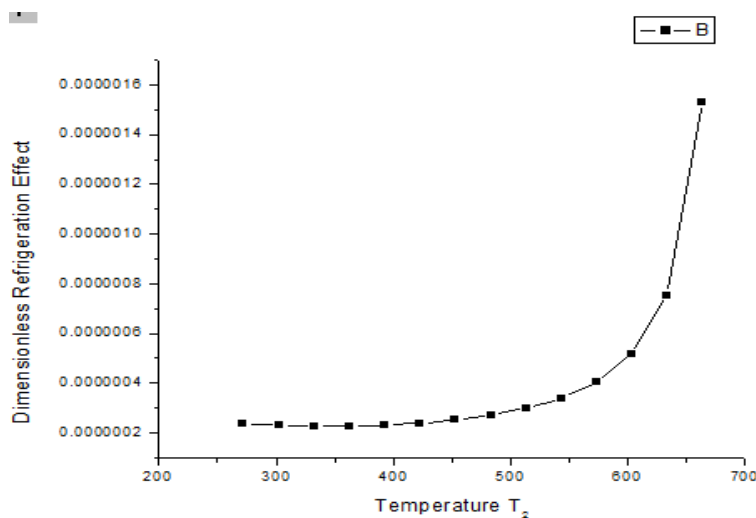


Figure 2 Variation of dimensionless Refrigeration Effect and temperature T_2

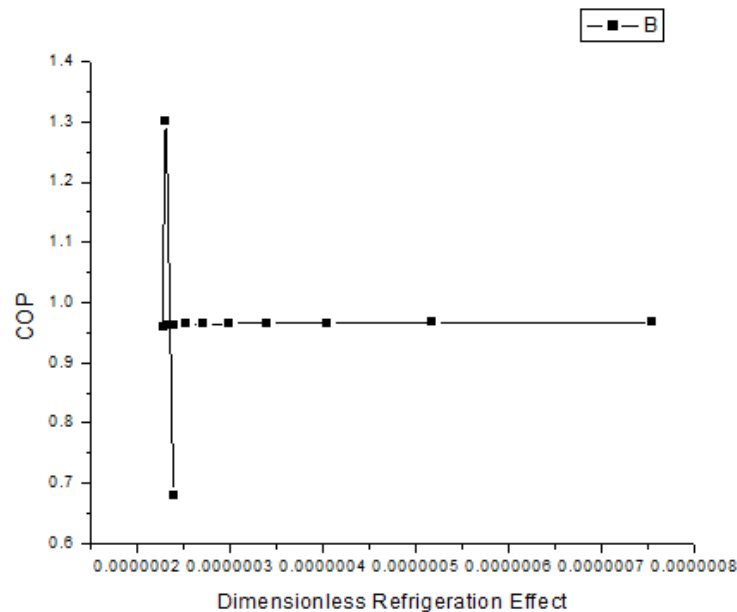


Figure 3 Variation of non dimensional term COP and temperature Refrigeration Effect

“Fig 1” Shows the variation of of non-dimensional term COP and temperature T_2 (K) where value of COP is maximum of 1.3 for the temperature of 330^0C . After which its value is decreasing and giving a constant pattern. As the temperature increased the COP also increased but after the temperature of 330^0C the COP started decreasing.

“Fig 2” Shows the variation of non-dimensional term refrigeration effect and temperature T_2 (K) where refrigeration effect is increasing hyperbolically with temperature.

“Fig 3” Shows variation of non dimensional term COP and refrigeration effect, which shows that after maximum COP of 1.3 the refrigeration effect is having a constant pattern. Up to the COP of 1.3 the refrigeration effect is increasing.

V CONCLUSION

With the help of the analysis of results we can conclude that as the temperature range is increasing the COP is increasing up to a certain value linearly then there is a drastic decrease in COP up to a constant value. Refrigeration effect is having a sudden increment in higher temperature range. Both of these statements suggest that for the better performance lower refrigeration effect will be there for the higher COP. It will a area of research in the future to find a way to increase the COP and Refrigeration Effect simultaneously.

VI ACKNOWLEDGEMENTS

We would like to acknowledge Curzon and Ahlborn, Wang Hao and Wu Guo-Xing whose research work helped us a lot in analyzing this Carnot cycle.

REFERENCES

1. E. Warburg, *Magnetische Untersuchungen u'ber einige Wirkungen der Koerzitivkraft*, Ann Phys, 13 (1881) 141–164.
2. P. Weiss, A. Piccard, *Sur un nouveau phe'nome`ne magne'tocalorique*, Compt Rend Ac Sci, 166 (1918) 352.
3. P. Debye, *Einige Bemerkungen zur Magnetisierung bei tiefer Temperatur*, Ann Phys, 81 (1926) 1154–1160.
4. W.F. Giauque, *A thermodynamic treatment of certain magnetic effects. A proposed method of producing temperatures considerably below 18 absolute*, J Am Chem Soc, 49 (1927) 1864–1870.
5. W.F. Giauque, D.P. MacDougall, *Attainment of temperatures below 18 absolute by demagnetization of Gd₂(SO₄)₃·8H₂O*, Phys Rev Lett 43 (9) (1933) 768.
6. Suxin Quian et al, *study of high efficient heat recovery cycle for solid state cooling*, international journal of refrigeration (2015) I 02 – I 19
7. Andrej Kitanovski, Peter W. Egolf, *Froid Magnetique*, International Journal of Refrigeration, 29 (2006) 3-21 .
8. Curzon F L and Ahlborn B, *Efficiency of a Carnot engine at maximum power output*, Am. J. Phys. 43(1975) 22.
9. Wang Hao and Wu Guo-Xing, *Ecological optimization for an irreversible magnetic Ericsson refrigeration cycle*. Chin.Phys. B, 22(2013) 8 87501 .
10. Pecharsky V K, Gschneidner Jr K A. *Giant Magnetocaloric effect in Gd₅(Si₂Ge₂)* [J]. Phys. Rev. Lett., 1997, 78:3299
11. Pecharsky V K, Gschneidner Jr K A. *Tunable magnetic regenerator alloys with a giant magnetocaloric effect for magnetic refrigeration from ~20 to ~290 K* [J]. Appl. Phys. Lett., 1997, 70:3299.